Temporal Entanglement, Quasiparticles, and the Role of Interactions

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In quantum many-body dynamics admitting a description in terms of *noninteracting* quasiparticles, the Feynman-Vernon influence matrix (IM), encoding the effect of the system on the evolution of its local subsystems, can be analyzed exactly. For discrete dynamics, the temporal entanglement (TE) of the corresponding IM satisfies an area law, suggesting the possibility of an efficient representation of the IM in terms of matrix-product states. A natural question is whether integrable interactions, preserving stable quasiparticles, affect the behavior of the TE. While a simple semiclassical picture suggests a sublinear growth in time, one can wonder whether interactions may lead to violations of the area law. We address this problem by analyzing quantum quenches in a family of discrete integrable dynamics corresponding to the real-time Trotterization of the *interacting* XXZ Heisenberg model. By means of an analytical solution at the dual-unitary point and numerical calculations for generic values of the system parameters, we provide evidence that, away from the noninteracting limit, the TE displays a *logarithmic* growth in time, thus violating the area law. Our findings highlight the nontrivial role of interactions, and raise interesting questions on the possibility to efficiently simulate the local dynamics of interacting integrable systems.

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While computing the exact properties of many-body quantum systems out of equilibrium remains a formidable problem, the past decades have witnessed the development of powerful numerical techniques allowing for accurate approximations. This is especially true in one dimension, where the dynamics can be simulated using algorithms based on matrix-product states (MPSs) [1–7]. Even in this case, however, the generic linear growth of the entanglement entropy [8,9] poses a major obstacle for the MPS representation of the time-evolving state [7].

When one is interested in the dynamics of local observables, it is natural to expect that much of the information encoded in the wave function is irrelevant, and that alternative approaches can be devised retaining only the data needed to reconstruct the local physics. A promising idea in this direction was put forward in Ref. [6] (see also Refs. [10–12]), which proposed an MPS algorithm to describe the dynamics induced on local subsystems. Crucially, the efficiency of the method is insensitive to the growth of the standard entanglement entropy. Instead, it is related to the so-called *temporal entanglement* (TE) [13], which is naturally understood as the entanglement generated along a space-time rotated direction [6]. This approach has recently received renewed interest in connection to the study of space-time dualities in Floquet kicked Ising chains [14,15] and dual-unitary quantum circuits [16], see Refs. [17–30]. In addition, similar ideas motivated related constructions exploiting space-time rotation in generic quantum-circuit dynamics [31–36].

Recently, the approach developed in Ref. [6] has been understood in more physical terms based on the so-called Feynman-Vernon influence matrix (IM) approach [13], where one views the system as an environment for its local subsystems. Complete information on the local dynamics is encoded in the IM, which can be thought of as a wave function in a multitime Hilbert space. The TE is the bipartite entanglement entropy of the IM.

For time-discrete evolution, it has been argued that the scaling of the TE provides valuable information about the

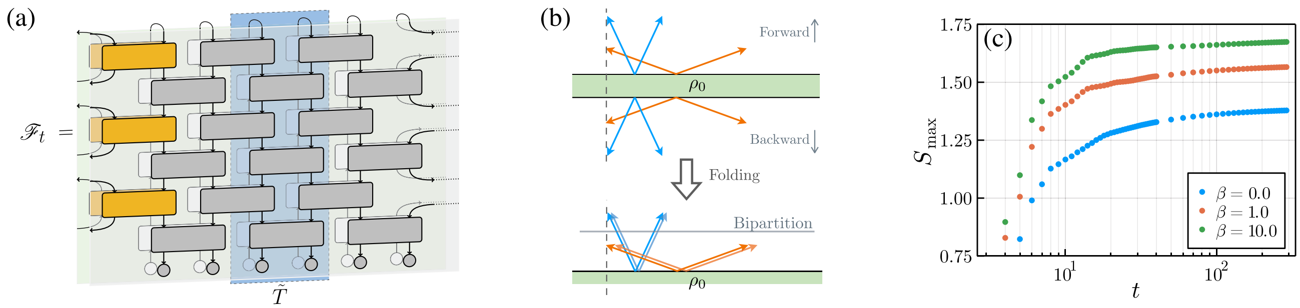

FIG. 1. (a) A system of L qubits is initialized in a product state and evolved via a brickwork quantum circuit, with two-site gate defined in Eq. (2). "Folding" the circuit, the left and right IMs determine the evolution of local subsystems. Fixing a site j, the Floquet operator decomposes into $\mathcal{U} = \mathcal{U}_{\rm int}\mathcal{U}_E$, cf. the main text. In the figure, $\mathcal{U}_{\rm int}$ consists of the highlighted gates, while the gray and white gates are part of $\mathcal{U}_E$. The operator $\tilde{T}$ defines the dual transfer matrix. (b) Cartoon of the quasiparticle picture for the TE. After folding, backward and forward world lines for each quasiparticle are superimposed, leading to the prediction of sublinear growth for the TE. (c) Growth of the TE for the noninteracting case. In the plot we set $J_x = 0.3, J_y = 0.5, J' = 0$, while the initial state is $\rho_0 = \bigotimes_k \rho_0^{(k)}$, with $\rho_0^{(k)} = e^{-\beta\sigma_k^z}/\mathcal{Z}$ and $\mathcal{Z} = 2\cosh\beta$.

nature of the dynamics [37–39], displaying, for instance, a slow growth in many-body localized phases [39]. Still, despite a few interesting examples [17,41–44], our understanding of the TE scaling remains largely incomplete.

As an exception, a detailed characterization of the TE was achieved for *noninteracting* systems, as exemplified for infinite-temperature states in the transverse-field kicked Ising chain [37]. Here, the IM was computed analytically, displaying a Bardeen-Cooper-Schrieffer-like structure, and the corresponding TE entropy was shown to obey an *area-law* scaling.

Since the analysis of Ref. [37] relies on a quasiparticle description, it is natural to ask about the fate of the TE area law in the presence of *integrable* interactions, preserving the stable quasiparticles. Besides its interest *per se*, this question has implications on the possibility to efficiently simulate the (discrete) dynamics of interacting integrable models, a task known to be hard from the analytical viewpoint [45–47].

We tackle this question by studying a family of dynamics corresponding to the Trotterization of the *interacting* XXZ Heisenberg model [48,49]. We focus on quenches from generic initial states, extending the setting of Ref. [37] to nonequilibrium situations. Based on a quasiparticle picture [8,50–54], we argue that the TE scaling is sublinear in time. We provide evidence that, while the area law is preserved for a large class of initial states in the noninteracting case, the TE exhibits a typical *logarithmic* growth in the presence of interactions, violating the area law. We conjecture this to be a generic feature of interacting integrable systems, and discuss some interesting questions raised by our results.

The model.—We consider a spin-1/2 chain with L sites and periodic boundary conditions. The discrete dynamics is driven by $\mathcal{U} = \mathcal{U}_{\rm odd}\mathcal{U}_{\rm even}$, with

$$\mathcal{U}_{\rm odd} = \prod_{n=1}^{L/2} U_{2n,2n+1}, \qquad \mathcal{U}_{\rm even} = \prod_{n=1}^{L/2} U_{2n-1,2n}, \qquad (1)$$

where

$$U_{n,n+1} = e^{-iJ_x\sigma_n^x\sigma_{n+1}^x - iJ_y\sigma_n^y\sigma_{n+1}^y - iJ'(\sigma_n^z\sigma_{n+1}^z - \mathbb{1})} \qquad (2)$$

is a two-site gate expressed in terms of Pauli matrices. We denote by $|0\rangle_j$, $|1\rangle_j$ the states in the local computational basis. Unless stated otherwise, we will set $J_x = J_y = J$. This model was introduced in Refs. [48,49] as a paradigmatic example of an integrable, periodically driven spin chain and can be thought of as a Trotterized XXZ Heisenberg evolution. These Floquet dynamics can be represented as a brickwork circuit, cf. Fig. 1(a).

For $J' = 0$, the model reduces to the XY spin chain, mappable to free-fermion dynamics by a Jordan-Wigner transformation, while for $J_x = J_y = \pi/4$ the circuit generated by repeated application of the Floquet operator $\mathcal{U}$ is dual unitary [16]. For arbitrary J, J', the system displays an extensive number of local conservation laws [48,49] and the Floquet spectrum may be obtained exactly via the Bethe ansatz [55,56]. The corresponding quasiparticle structure bears similarities to that of the well-known XXZ Heisenberg Hamiltonian [55,57].

The quench protocol and the IM.—We study a quench, where the system is initialized in product states (either pure or mixed), and analyze the subsequent evolution in the thermodynamic limit $L \to \infty$. The IM formalism [13] may be introduced starting from the time-evolved expectation value ${\rm Tr}[\rho(t)O_j] = {\rm Tr}[\rho_0(\mathcal{U}^\dagger)^t O_j \mathcal{U}^t]$ of a local observable O_j at site j. Taking for simplicity an initial state $\rho_0 = \otimes_k \rho_0^{(k)}$, the parts of the system to the left and right of j will be treated as *environments*. The IMs associated with them arise from integrating out the environment degrees of freedom, treating the trajectory of spin j as an external parameter. Focusing on the right environment $k > j$, we can write down the IM as a Keldysh path integral, where forward and backward spin trajectories are "folded" on a closed time contour. We introduce a subsystem-environment decomposition $\mathcal{U} = \mathcal{U}_{\rm int}\mathcal{U}_E$, where $\mathcal{U}_{\rm int}$ is the gate acting on spins j and $j+1$, and $\mathcal{U}_E$ acts only on spins $k > j$, which can be done in a natural way exploiting the brickwork structure, cf. Fig. 1(a). Defining the partial matrix elements of $\mathcal{U}_{\rm int}$ as the operators $[\mathcal{U}_{\rm int}]_{s,\sigma} = [U_{j,j+1}]^s_\sigma$ acting on spin $j+1$ only (where s, σ are the input and output states of spin j), the IM $|\mathscr{F}_t\rangle$ is the vector with coordinates depending on the trajectories $\{s^\pm_\tau, \sigma^\pm_\tau\}$ as

$$\mathscr{F}_t[\sigma^\pm_\tau, s^\pm_\tau] = \mathrm{Tr}_E([\mathcal{U}_{\rm int}]_{s^+_t,\sigma^+_t}\mathcal{U}_E\ldots\mathcal{U}_E[\mathcal{U}_{\rm int}]_{s^+_1,\sigma^+_1} \times \mathcal{U}_E\rho^E_0\mathcal{U}^\dagger_E[\mathcal{U}^\dagger_{\rm int}]_{\sigma^-_1,s^-_1}\mathcal{U}^\dagger_E\ldots\mathcal{U}^\dagger_E[\mathcal{U}^\dagger_{\rm int}]_{\sigma^-_t,s^-_t}), \quad (3)$$

where $\mathrm{Tr}_E \equiv \mathrm{Tr}_{k>j}$ and $\rho^E_0 \equiv \bigotimes_{k>j}\rho^{(k)}_0$, cf. Fig. 1.

The IM of a longer environment can be computed from that of a shorter one, leading to an exact self-consistency equation in the thermodynamic limit [6,13]. As depicted in Fig. 1, this can be formalized by introducing a dual transfer matrix $\tilde{T}$ generating the evolution in a "rotated direction": the self-consistency equation reads $\tilde{T}|\mathscr{F}_t\rangle = |\mathscr{F}_t\rangle$ [6,10,13] and completely determines $|\mathscr{F}_t\rangle$.

The TE and the quasiparticle picture.—The quantity of interest in this Letter is the TE entropy, denoted by $S_\tau(t)$. In order to define it, we consider a bipartition of the multitime Hilbert space of spin trajectories, cut into two regions with time labels $0 \le t' \le \tau$ and $\tau + 1/2 \le t'' \le t$. Here $t', t'' \in (0, t)$ are half integers. The TE is the von Neumann entanglement entropy [58] of the state $|\mathscr{F}_t\rangle$ associated with this bipartition.

We recall that the growth of the standard entanglement entropy after a quantum quench in integrable systems is captured by a well-known quasiparticle picture [8,50–54]. In essence, one postulates that the quench can be modeled as a process creating at each point in space uncorrelated pairs of entangled quasiparticles spreading through the system with opposite momenta. Given two disjoint regions A and B, their entanglement then grows proportionally to the number of pairs with one quasiparticle in A and the other in B. When supplemented with model-dependent data, this results in a quantitative prediction for the linear growth of the entanglement entropy, which has been proved analytically for noninteracting chains [50] and extensively tested numerically in interacting models [53,54,59–61].

Heuristically, we may apply this picture to the TE, cf. Fig. 1(b). Now for each pair we have to keep track of both the forward and backward evolution. Although these trajectories are correlated, they end up being superimposed in the folded spacetime. As a consequence, given a "space slice," all correlated quasiparticles occupy the same temporal position on the Keldysh contour and quasiparticles are not able to transport entanglement at different time sites. One concludes that no TE is generated between disjoint temporal regions after the quench [62].

A similar heuristic argument already appeared in Ref. [6]. However, there it was stated in terms of noninteracting localized excitations and supported by the analysis of a circuit of swap gates [10]. In contrast, we insist that the picture presented here is in terms of the stable collective quasiparticles of integrable models. As such, it is expected to hold in the scaling limit of large times and to only provide predictions for the leading-order behavior of the TE. That is, the above argument suggests that the TE in integrable systems must asymptotically grow *sublinearly* in time.

This prediction is consistent with the TE area law scaling found in Ref. [37] for the infinite-temperature kicked Ising chain

$$\max_\tau[S_\tau(t)] \le c, \quad \forall\ t, \quad (4)$$

where c is a constant. This result was derived by mapping the system to noninteracting fermions and constructing a gapped quasilocal parent Hamiltonian for $|\mathscr{F}_t\rangle$. The area law (4) has been numerically confirmed exploiting a covariance-matrix approach for efficient evaluation. A similar analysis can be carried out in our model for $J' = 0$, for which Eq. (1) is mapped onto a free-fermion evolution. In addition, although Eq. (4) was originally shown for infinite-temperature states [37], the covariance-matrix approach can be generalized to any *Gaussian* initial state [67], allowing us to confirm Eq. (4) for different values of J_x, J_y, and various quenches. An example of our data is shown in Fig. 1(c).

Next, our goal is to test the prediction of the quasiparticle picture and scaling (4) in the presence of interactions. We provide evidence that, while the TE growth is indeed sublinearly in time, interactions bring about logarithmic violations of the area law.

Exact IM at the dual-unitary point.—In principle, integrability allows one to diagonalize the rotated transfer matrix $\tilde{T}$ via the Bethe ansatz and obtain an explicit expression for the IM [72,73]. However, the resulting wave function is too complicated, and it is not known how to extract the corresponding entanglement.

In order to get some analytical insight, we consider $J = \pi/4$, for which the dynamics is dual unitary [16]. While in this case the TE is vanishing for a class of fine-tuned initial states [17], here we are interested in its behavior for generic ones. To be concrete, we consider a product state $|\Psi_0\rangle = |+\rangle^{\otimes L}$, with $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, although our results generalize to arbitrary two-site shift invariant states $|\Psi_0\rangle = |\psi\rangle_{1,2} \otimes |\psi\rangle_{3,4} \otimes \cdots \otimes |\psi\rangle_{L-1,L}$.

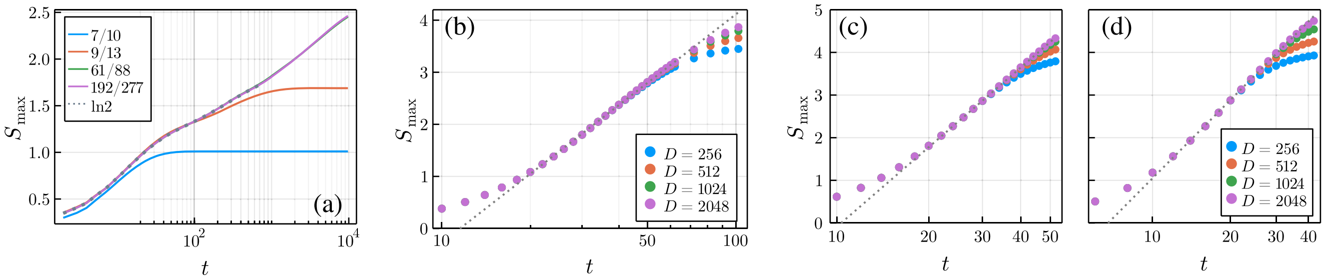

FIG. 2. Maximum TE $S_{\max} = \max_\tau[S_\tau(t)]$ as a function of time for different values of $J_x = J_y = \pi/4 + \epsilon$ and $J' = \pi/4 + K$. (a) TE at the dual-unitary point $\epsilon = 0$ and $K/\pi = \ln(2)$, quenching from $|\Psi_0\rangle = |+\rangle^{\otimes L}$. The plot is obtained evaluating the entanglement entropy of the analytical MPS solution (5). We also show the TE for rational values of K/π approximating ln(2). (b) TE at $\epsilon = 0.05$, $J' = 1$ for the infinite-temperature state. (c)–(d) Same plot for the Néel state $|\Psi_0\rangle = |01\rangle^{\otimes L/2}$. The parameters are $J' = 1$ and $\epsilon = 0.05$ (c), $\epsilon = 0.08$ (d) Dotted lines are a guide to the eye to emphasize the logarithmic growth.

The Bethe ansatz description remains nontrivial at $J = \pi/4$ [55]. Nonetheless, the form of the gate in Eq. (2) becomes simple, allowing us to obtain an exact MPS expression for the IM. Interestingly, we do this avoiding Bethe ansatz techniques and relying instead on methods borrowed from analytical tensor-network theory [43,68]. We consign the details to the Supplemental Material [67], while here we simply report the final result of our analysis. Setting $J' = \pi/4 + K$, we find that the left IM is

$$\langle \mathscr{F}_t| = \langle v|A^{[1]}B^{[2]}A^{[3]}B^{[4]}\ldots A^{[2t-1]}|w\rangle. \qquad (5)$$

Here, A, B are tensors with four physical indices labeled by 00,01,10,11, and bond dimension $2t+1$. The corresponding matrices are defined by the elements $[A_{00}]_{\alpha,\beta} = \delta_{\alpha,\beta}\cos[2K\alpha]$, $[A_{01}]_{\alpha,\beta} = \delta_{1,\alpha-\beta}\cos[2K(\alpha-1)]$, $[A_{10}]_{\alpha,\beta} = \delta_{1,\beta-\alpha}\cos[2K\beta]$, $[A_{11}]_{\alpha,\beta} = [A_{00}]_{\alpha,\beta}$, and $[B_{00}]_{\alpha,\beta} = \delta_{\alpha,\beta}\exp[2Ki\alpha]$, $[B_{11}]_{\alpha,\beta} = \delta_{\alpha,\beta}\exp[-2Ki\alpha]$, $[B_{01}]_{\alpha,\beta} = [B_{10}]_{\alpha,\beta} = 0$. Here $\alpha,\beta = -t, -(t-1), \ldots, t$. In addition, the boundary vectors are defined by the elements $|v\rangle_\alpha = \delta_{\alpha,0}$ and $|w\rangle_\alpha = 1$. A similar expression holds for the right IM [67].

As an immediate consequence, we obtain

$$\max_\tau[S_\tau(t)] \leq \ln(2t+1) \sim \ln(t), \qquad (6)$$

yielding a rigorous proof for the sublinear growth of the TE. Here we used that the bipartite entanglement entropy of an MPS with bond dimension D is bounded by $\ln D$ [7]. Despite the simplicity of the solution, the TE displays interesting features. First, we find that the asymptotic behavior at large times is not continuous as a function of K. In order to see this, we take $K = (n\pi/m)$, with n, m coprime integers. In this case, it is easy to see that the MPS (5) can be *compressed* to one with finite bond dimension: because of the periodicity of the trigonometric functions, the infinite matrices $A^{[i]}$ and $B^{[i]}$ can be truncated to the first m lines and columns, so that the TE is bounded. However, this compression is not possible when K/π is irrational, suggesting a logarithmic growth.

In order to verify this, we evaluated numerically the TE for irrational values of K/π, which can be done efficiently since the MPS form of the IM is known exactly. An example of our data is reported in Fig. 2(a), providing evidence of a logarithmic growth. We also show the TE corresponding to rational values approximating K/π [74]. In other cases, we observe that the TE might display extremely long initial plateaux, which we attribute to the vicinity of K/π to rational numbers with small denominator, see Ref. [67]. This, in general, makes it challenging to extrapolate the asymptotic behavior from finite-time data. Therefore, while the upper bound (6) is rigorous, given the highly irregular behavior of the TE at the dual unitary point, our numerical evidence should be taken *cum grano salis* in this case. Still, for the accessible timescales, our data consistently point to an indefinite growth of the TE for generic J, thus violating the area law [75].

Numerical study for generic interactions.—Away from the dual-unitary point, the IM may be obtained using MPS numerical methods. Following Refs. [6,10,13], we represent $\tilde{T}$ as a matrix product operator (MPO), and compute its leading eigenvector using either the density-matrix renormalization group (DMRG) [7], or power methods [6]. In order to push the available simulation times, we focus on initial states displaying $U(1)$ symmetry, allowing us to enforce it explicitly in the local tensors [76]. Finally, throughout our simulations we used the bond dimension D as a control parameter, checking convergence with respect to it.

We first consider the infinite-temperature state for different values of J, J'. Away from $J = \pi/4$, we find no evidence of a TE area law for rational values of K/π. In general, we observe an initial linear increase of the TE, followed by an eventual logarithmic growth. Our data are shown in Fig. 2(b): for the available times, curves corresponding to increasing D are seen to converge to a straight line in logarithmic scales.

Next, we turn to the TE from non-equilibrium initial states. In order to preserve U(1) symmetry, we have chosen the Néel state $|\Psi_0\rangle = |01\rangle^{\otimes L/2}$. Here we observe that the TE is large compared to the infinite-temperature state and increasing as we move away from the dual-unitary point. The TE is not symmetric around $t/2$, with its maximum generally attained at later times between $t/2$ and t. Its precise location varies with the initial states and parameters. Our numerical data are shown in Fig. 2. Although simulation times are limited, we observe a convincing logarithmic growth emerging after an initial short-time regime. Altogether, our results consistently point to a typical logarithmic violation of the area law in the presence of interactions, which we conjecture to be a general feature of interacting integrable systems.

Outlook.—We have studied the TE in integrable discrete dynamics. Starting from a heuristic quasiparticle picture and based on analytical and numerical evidence in the XXZ Heisenberg model, we have put forward that the TE generically grows logarithmically in time, violating the area law scaling away from the noninteracting regime.

Our findings raise several questions. First, it would be interesting to put our conjecture on rigorous grounds away from the dual-unitary point. From the computational point of view, instead, it would be important to understand whether and how the sublinear growth of the TE may be exploited for an efficient computation of the IM and its approximation in terms of MPS.

A natural question pertains to the relation between the growth of the TE and the operator-space entanglement entropy (OSEE) of local observables [77,78]. In fact, the latter was also conjectured to grow logarithmically in integrable systems [78–80]; see Refs. [21,22,81–83] for a proof in special cases. However, at the dual-unitary point of Eq. (2), the OSEE was shown to satisfy an area law [22]. Therefore, our results suggest that the OSEE is not directly related to the TE: this is consistent with the intuition that the IM bears information beyond the Heisenberg evolution of local observables, see, e.g., Refs. [42,43].

Finally, while we have focused on discrete dynamics, it would be interesting to study the Trotter limit of continuous-time evolution. Preliminary results suggest that the TE could be *vanishing* in this limit, similarly to the noninteracting case studied in Ref. [37]. This would indicate that a continuous MPS ansatz [84,85] could be successfully employed in this limit.

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