Sizeable θ_{13} from the charged lepton sector in SU(5), (tri-)bimaximal neutrino mixing and Dirac CP violation

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ABSTRACT: The recent results from T2K and MINOS experiments point towards a relatively large value of the reactor angle θ_{13} in the lepton sector. In this paper we show how a large θ_{13} can arise from the charged lepton sector alone in the context of an SU(5) GUT. In such a scenario (tri-)bimaximal mixing in the neutrino sector is still a viable possibility. We also analyse the general implications of the considered scenario for the searches of CP violation in neutrino oscillations.

Contents

1 Introduction

Recently the T2K collaboration reported [1] evidence at 2.5σ for a non-zero value of the reactor angle θ_{13} in the neutrino mixing matrix. Quantitatively, it was found that $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ at 90% C.L. for $\Delta m_{32}^2 = {}^{+}_{(-)} 2.4 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2 2\theta_{23} = 1$ and $\delta = 0$, Δm_{32}^2 and θ_{23} being the atmospheric neutrino mass squared difference and mixing angle, and δ being the Dirac CP violating phase in the neutrino mixing matrix (see, e.g. [2]). Under the same conditions the best fit value of θ_{13} obtained from the data analysis is relatively large: $\sin^2 2\theta_{13} = 0.11$ (0.14). Subsequently the MINOS collaboration also reported evidence for a relatively large value of θ_{13}, although with a smaller statistical significance [3]. A global analysis of the neutrino oscillation data, including the data from the T2K and MINOS experiments, performed in [4] showed that actually $\sin\theta_{13} \neq 0$ at $\geq 3\sigma$. The authors of [4] find:

$$\sin^2\theta_{13} = 0.021 \ (0.025) \pm 0.007 \,, \tag{1.1}$$

using the "old" ("new") fluxes of reactor $\bar\nu_e$ in the analysis. Moreover, it was found in the same global analysis that $\cos\delta = -1$ (and $\sin\theta_{13}\cos\delta = -0.14$) is clearly favored by the data over $\cos\delta = +1$ (and $\sin\theta_{13}\cos\delta = +0.14$[1]).

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[1] This result can have important implications for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe [5].

The T2K and MINOS results will be tested in the upcoming reactor neutrino experiments Double Chooz [6], Daya Bay [7] and RENO [8]. If confirmed, they will have far reaching implications for the program of future research in neutrino physics. A relatively large value of θ_{13} opens up the possibilities, in particular, i) for searching for CP violation effects in neutrino oscillations experiments with high intensity accelerator neutrino beams (like T2K, NOνA, etc.); ii) for determining the sign of Δm_{32}^2, and thus the type of neutrino mass spectrum, which can be normally or invertedly ordered (see, e.g. [2]), in the long baseline neutrino oscillation experiments at accelerators (NOνA, etc.), in the experiments studying the oscillations of atmospheric neutrinos (see, e.g. [9–11]), as well as in experiments with reactor antineutrinos [12–14]. A value of $\sin\theta_{13} \gtrsim 0.09$ is a necessary condition for a successful "flavoured" leptogenesis when the CP violation required for the generation of the matter-antimatter asymmetry of the Universe is provided entirely by the Dirac CP violating phase in the neutrino mixing matrix [15, 16].

In the present article we investigate the possibilities to obtain the relatively large value of θ_{13}, suggested by the T2K, MINOS and the current global neutrino oscillation data, in the case when the diagonalization of the neutrino mass matrix gives a negligible contribution to θ_{13}. Parametrically the latter means a contribution smaller than $\mathcal{O}(\lambda^2)$, where $\lambda = 0.23$ is the Cabibbo angle expansion parameter. The diagonalization of the neutrino mass matrix leads to a negligible θ_{13}, for example, in models with $L_e - L_\mu - L_\tau$ flavour symmetry [17] and in most models accounting for bimaximal (BM) [18–22] or tri-bimaximal (TBM) [23–27] neutrino mixing. For our purposes, this is equivalent to setting the neutrino mass matrix contribution to θ_{13} to zero, which we will do in our further analysis. The θ_{13} mixing angle can then be due to the contribution to the neutrino mixing coming from the diagonalization of the charged lepton mass matrix (see, e.g. [28–31] and the references quoted therein[2]).

In particular, we will consider the possibility of a "large" θ_{13} originating from the charged lepton mass matrix in the context of SU(5) supersymmetric unification. The motivation for such a choice are strong and well known. In our case, this choice will also allow to get a handle on the expected contribution to θ_{13} and on the structure of the charged lepton Yukawa coupling matrix. This is because the SU(5) symmetry allows to relate the latter and the down quark one, on which further experimental information is available.

In minimal SU(5) models, the down-quark and transposed charged lepton mass matrices are equal. On the other hand, depending on the (renormalizable or non-renormalizable) SU(5) operator(s) giving rise to a given Yukawa coupling matrix entry, the relation may involve Clebsch-Gordan (CG) coefficients. Such coefficients need to be invoked in order to fix the SU(5) predictions for the m_b/m_τ and m_s/m_μ mass ratios (unless specific values of $\tan\beta$, peculiar patterns of SUSY breaking parameters, or highly asymmetric mass textures are employed [37–43]), as shown by the authors of [44, 45] after having identified the possible GUT scale mass ratios in a bottom-up approach. All the possible CG coefficients in SU(5) and the Pati-Salam group entering such relations arising from dimension five and

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[2]For alternative possibilities leading to a relatively large θ_{13}, see [32] as well as, e.g. [33–36].

some dimension six operators are listed in [45]. Such coefficients not only can give rise to new mass ratios, but can also affect the prediction for θ_{13} [45]. In this paper, we discuss possible combinations of CG coefficients, which can give a "large" θ_{13} even if there is no contribution to θ_{13} from the neutrino mass matrix at all.

The paper is organised as follows. In section 2 we describe the general set up, the determination of the standard neutrino parameters under the assumptions we make, the numerical procedure we used to determine the allowed CG coefficients and the predictions for θ_{13}. The results are illustrated in section 3. In section 4 we consider the special case in which the neutrino contribution to the PMNS matrix is of the BM or TBM form, in which interesting implications for the leptonic (Dirac) CP-violation phase can be drawn. In section 5 we summarize.

2 General setup and procedure

2.1 The setup

We start with some definitions and a summary of the assumptions we make regarding the structure of the mixing matrices. Thereby we follow closely the discussion in [28–31]. The PMNS lepton mixing matrix U is given by

$$U = U_e U_\nu^\dagger, \quad \text{with } U_e,\, U_\nu \text{ defined by} \quad \begin{aligned} m_E &= U_{e^c}^T m_E^{\text{diag}} U_e, \\ m_\nu &= U_\nu^T m_\nu^{\text{diag}} U_\nu \end{aligned}, \tag{2.1}$$

where m_E and m_ν are the charged lepton and light neutrino mass matrices, respectively, and m_E^{diag}, m_ν^{diag} are diagonal and positive. The PMNS matrix can be parameterized in the standard way as $U = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) Q$, where Q is a diagonal phase matrix containing the two physical Majorana CP violation phases [46–48].

Barring correlations among the entries of m_E, the hierarchy of the charged lepton masses translates into the possibility of diagonalizing m_E perturbatively through subsequent 2×2 unitary rotations in the following order: $U_e = U_{12}^e U_{13}^e U_{23}^e$, where U_{ij}^e is a unitary rotation in the ij block. The 13 rotation is negligible for our purposes and will be set to zero, thus $U_{13}^e = \mathbf{1}$.

The assumption of zero neutrino mass matrix contribution to θ_{13} can be rephrased as $\theta_{13} = 0$ in the limit of $U_e = \mathbf{1}$, or equivalently as $U_\nu^\dagger = U|_{U^e=\mathbf{1}} = R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu)$ up to phase matrices. Note that since U_{23}^e does not contribute to θ_{13}, the assumption can be further rephrased as

$$U = R_{12}(\theta_{12}^e) \Phi R_{23}(\hat{\theta}_{23}) R_{12}(\hat{\theta}_{12}) Q$$

$$= \begin{pmatrix} \hat{c}_{12} c_{12}^e - \hat{s}_{12} \hat{c}_{23} s_{12}^e e^{i\phi} & \hat{s}_{12} c_{12}^e + \hat{c}_{12} \hat{c}_{23} s_{12}^e e^{i\phi} & \hat{s}_{23} s_{12}^e e^{i\phi} \\ -\hat{c}_{12} s_{12}^e - \hat{s}_{12} \hat{c}_{23} c_{12}^e e^{i\phi} & \hat{c}_{12} \hat{c}_{23} c_{12}^e e^{i\phi} - \hat{s}_{12} s_{12}^e & \hat{s}_{23} c_{12}^e e^{i\phi} \\ \hat{s}_{12} \hat{s}_{23} & -\hat{c}_{12} \hat{s}_{23} & \hat{c}_{23} \end{pmatrix} Q. \tag{2.2}$$

Here $\Phi = \text{Diag}(1, e^{i\phi}, 1)$, where ϕ is a CP-violating phase [28, 29]. The angles θ_{12}^e, $\hat{\theta}_{12}$, $\hat{\theta}_{23}$ and the phase ϕ in the expression (2.2) for U can be arranged to lie in the intervals $[0, \pi/2]$ and $[0, 2\pi]$, respectively.

By commuting with the $\hat\theta_{23}$ one, the θ_{12}^e-rotation has induced a non vanishing θ_{13} given by

$$\sin\theta_{13} = |U_{e3}| = \frac{\sin\theta_{12}^e \tan\theta_{23}}{\sqrt{1 + \tan^2\theta_{23} - \sin^2\theta_{12}^e}} \approx \sin\theta_{12}^e \sin\theta_{23}\,. \tag{2.3}$$

Note that we have traded here $\hat\theta_{23}$ with θ_{23} in the standard parameterization. Note also that the corrections to $\hat\theta_{23}$ and $\hat\theta_{12}$ induced by θ_{12}^e are given by

$$\tan\theta_{23} = \cos\theta_{12}^e \tan\hat\theta_{23} \tag{2.4a}$$

$$\tan\theta_{12} = \tan\hat\theta_{12} \left| \frac{1 + \cos\hat\theta_{23} \tan\theta_{12}^e e^{i\phi}}{1 - \tan\hat\theta_{12} \cos\hat\theta_{23} \tan\theta_{12}^e e^{i\phi}} \right|\,. \tag{2.4b}$$

In the case of 3-ν mixing under discussion, the magnitude of the CP violation effects in neutrino oscillations is determined [49] by the rephasing invariant associated with the Dirac CP violating phase δ:

$$J_{\mathrm{CP}} = \mathrm{Im}\big\{U_{e1}^* U_{\mu3}^* U_{e3} U_{\mu1}\big\}\,. \tag{2.5}$$

The rephasing invariant J_{CP}, as is well known, is a directly observable quantity. It is analogous to the rephasing invariant associated with the Dirac phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix, introduced in [50, 51]. In the standard parametrisation of the PMNS matrix we find

$$J_{\mathrm{CP}} = \frac{1}{8} \sin2\theta_{23} \sin2\theta_{12} \cos\theta_{13} \sin2\theta_{13} \sin\delta\,. \tag{2.6}$$

Using eqs. (2.5) and (2.2) we get [31]

$$J_{\mathrm{CP}} = -\frac{1}{8} \sin2\theta_{12}^e \sin2\hat\theta_{12} \sin2\hat\theta_{23} \sin\hat\theta_{23} \sin\phi\,. \tag{2.7}$$

Thus, to leading order in $\sin\theta_{12}^e$ we have $\sin\delta = -\sin\phi$. By comparing the real part of $U_{e1}^* U_{\mu3}^* U_{e3} U_{\mu1}$ in the two parameterisations we conclude that, at the same order,

$$\delta = -\phi\,. \tag{2.8}$$

2.2 Relation between θ_{12}^e and θ_{13} in GUTs

We would like now to study the possibility to generate a θ_{12}^e large enough to induce a θ_{13} in the range indicated by recent experiments in the context of an SU(5) Grand Unified Theory (GUT). The unification assumption is powerful because it allows to relate the charged lepton and down quark Yukawa matrices λ_E and λ_D. If all the Yukawa entries were generated by renormalizable operators and the MSSM Higgs fields were embedded in **5** and $\bar{\mathbf{5}}$ representations only, we would have $\lambda_{ji}^E = \lambda_{ij}^D$, leading to wrong predictions for the fermion mass ratios. In the general case one has instead $\lambda_{ji}^E = \alpha_{ij}\lambda_{ij}^D$. The coefficients α_{ij} depend on the operators from which the Yukawa entries arise. Such values can be constrained to belong to a finite set of rational numbers at the price of assuming that each Yukawa entry comes at least dominantly from a single renormalizable or non-renormalizable

Operator Dimension	α_{ij}
4	1
	-3
5	$-1/2$
	1
	$\pm 3/2$
	-3
	$9/2$
	6
	9
	-18

Table 1. Summary of possible SU(5) predictions for the coefficients α_{ij}. Numbers are taken from [45], where also the corresponding operators are listed.

SU(5) operator.[3] In this case, the possible values of the α_{ij} coefficients are listed in table 1, see also [45].

The θ_{12}^e angle is obtained from the diagonalization of the 12 block of the charged lepton Yukawa matrix after the 23 block has been diagonalized. Let us denote such 12 blocks in the charged lepton and down quark sectors (in the RL convention in which the Yukawa interactions are written with the left-handed fields on the right) as

$$\hat{\lambda}_{[12]}^D = \begin{pmatrix} a & b' \\ b & c \end{pmatrix} \qquad \hat{\lambda}_{[12]}^E = \begin{pmatrix} \alpha a & \beta b \\ \beta' b' & \gamma c \end{pmatrix}. \tag{2.9}$$

In the following we will assume that the entries in eq. (2.9) can be approximated with the corresponding entries of $\lambda^{E,D}$, in which case the coefficients α, β, β', γ are still bound to take one of the values in table 1 (the rotation used to diagonalize the 23 sector can have a sizeable effect on the coefficient γ and, if the charged lepton contribution to θ_{23} from U_{23}^e is sizeable, on the coefficient β).

We would like to determine the values of the coefficients α, β, β', γ allowed by data, and in particular capable to account for the indication for a sizeable θ_{13}. Not all the values of the coefficients are allowed, in principle. The observables to be described are in fact

$$\theta_{13}, \quad |V_{us}|, \quad \frac{m_e}{m_\mu}, \quad \frac{m_d}{m_s}, \quad \frac{m_\mu}{m_s}, \tag{2.10}$$

and for given α, β, β', γ, the five experimental inputs above depend on the four real variables $|b/c|$, $|b'/c|$, $|a/c|$, ω, where the phase ω is defined by $ac(bb')^* = e^{i\omega}|acbb'|$. The

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[3]This could not be the case, for example, if SU(5) is embedded in SO(10) or a larger unified group.

Input Parameter	Value	Assumed error distribution
m_e/m_μ	$(4.7362 - 4.7369) \times 10^{-3}$ [57]	Uniform
m_μ/m_s	$2.48 - 7.73$ [45]	Uniform
m_s/m_d	18.9 ± 0.8 [58]	Gaussian
$|V_{us}|$	0.2252 ± 0.0009 [59]	Used uniform in $|V_{us}| \pm \Delta$
$\sin\theta_{13}$	$0.089 - 0.190$ (2σ) [4]	Approximated with a Gaussian

Table 2. List of input parameters used in our analysis.

explicit dependence is given by the following relations

$$\tan\theta_{12}^e = \left| \frac{\beta'b'}{\gamma c}\left(1 - \left|\frac{\beta b}{\gamma c}\right|^2\right) + \frac{\beta b^*}{\gamma c^*}\frac{\alpha a}{\gamma c}\right| \tag{2.11a}$$

$$|V_{us}| = \left| \frac{b}{c}\left(1 - \left|\frac{b'}{c}\right|^2 - \frac{1}{2}\left|\frac{b}{c}\right|^2\right) + \frac{b'^*}{c^*}\frac{a}{c}\right| \pm \Delta \tag{2.11b}$$

$$\frac{m_e}{m_\mu} = \left|\frac{\alpha}{\gamma}\frac{a}{c} - \frac{\beta\beta'}{\gamma^2}\frac{bb'}{c^2}\right|\left(1 - \frac{\beta^2|b|^2 + \beta'^2|b'|^2}{\gamma^2|c|^2}\right) \tag{2.11c}$$

$$\frac{m_d}{m_s} = \left|\frac{a}{c} - \frac{bb'}{c^2}\right|\left(1 - \frac{|b|^2 + |b'|^2}{|c|^2}\right) \tag{2.11d}$$

$$\frac{m_\mu}{m_s} = |\gamma|\left(1 + \frac{(\beta^2 - \gamma^2)|b|^2 + (\beta'^2 - \gamma^2)|b'|^2}{2|c|^2\gamma^2}\right), \tag{2.11e}$$

where Δ takes into account the possibility of a model-dependent contribution to $|V_{us}|$ from the up quark sector and is assumed to be in the range $|\Delta| < \sqrt{m_u/m_c} \approx 0.045$. The experimental inputs used for the quantities on the l.h.s. are listed in table 2. The relations above are approximated and are accurate up to corrections of order λ^4, if $|b/c| \lesssim |b'/c| \lesssim \lambda$, $|a/c| \lesssim \lambda^2$.

Besides the general case in eq. (2.9), we will also consider the case in which $a = 0$ and the symmetric case in which $|\lambda_{12}^D| = |\lambda_{21}^D|$ and $|\lambda_{12}^E| = |\lambda_{21}^E|$, as they arise in many models of fermion masses. Note that the symmetry condition implies $b = \pm b'$ and $\beta = \beta'$.

2.3 Procedure

Before we come to the results we briefly discuss the procedure we implemented. Since only the ratios $|a/c|$, $|b/c|$, and $|b'/c|$ enter when computing the experimental inputs, we have set in our forthcoming numerical analysis $|c| = 1$. We can also always perform a phase redefinition of the fields such that all the remaining coefficients are real and positive and the only physical phase is in a, so that $a = \exp(i\omega)$.

For each possible combination of CG coefficients we diagonalized exactly both the mass matrices, using the expressions for the observables in eq. (2.10) in terms of a, b, b', ω, of which the relations in eq. (2.11) are the expansion at NLO. Then we determined numerically a solution for these parameters such that all the experimental inputs are satisfied. We extracted the values for these inputs randomly following the distributions given in table 2. We repeated this procedure until one solution is found. If, after a large number of attempts,

$\{\alpha, \beta, \beta', \gamma\}$	$\{a, b, b', \omega\}$	$\sin\theta_{13}$
$\{-3, 1, -3, -3\}$	$\{0.0151, 0.220, 0.189, -2.81\}$	0.130 ± 0.013

Table 3. Possible CG coefficients with Yukawa couplings coming only from renormalizable operators. We also show typical values for the entries of $\hat{\lambda}_D$, where c is normalised to one, and we give the prediction for $\sin\theta_{13}$ inlcuding its 1σ standard deviation.

no solution is found, we discard this combination of CG coefficients. For the viable CG coefficients we obtained by this procedure a distribution for θ_{13}, from which we computed the mean value and the standard deviation. To obtain $\sin\theta_{13}$ from $\sin\theta_{12}^e$ we have assumed that θ_{23} in the neutrino sector is maximal for simplicity. Given the uncertainties on the other input variables, this is a good approximation.

Note that eq. (2.11e) fixes γ to lie in the range of the observed m_μ/m_s. Therefore we used this equation only to reduce the possible values of γ to -3, $9/2$ and 6, cf. table 2. The GUT scale ratio m_μ/m_s depends strongly on low energy SUSY threshold corrections [52–55] and in principle one can use them to push this ratio to more extreme values, but in simple SUSY breaking scenarios these are the only plausible values [45].

3 Results

In the most general case it is easy to obtain values of the CG coefficients leading to a value of $\sin\theta_{13}$ compatible with the recent fits in [4]. In fact we find several hundred possible combinations. We therefore restrict ourselves in the following to some well motivated cases. We give a graphical summary of the results at the end of this section in figure 1.

3.1 Results for renormalizable operators only

We start our discussion with the case in which the Yukawa couplings come only from renormalizable operators. This case is very restrictive as there are only two possible CG coefficients, which are $\alpha_{ij} = 1$, if the Higgs sits in a $\bar{\mathbf{5}}$ of SU(5), and $\alpha_{ij} = -3$, if the Higgs sits in a $\overline{\mathbf{45}}$ of SU(5) [56].

There is only one combination which is in agreement with the experimental data. It is shown in table 3, where we give in addition typical values for the entries of $\hat{\lambda}_D$ and the prediction for $\sin\theta_{13}$.

3.2 Results without representations larger than the adjoint

The next case we consider is the one in which the Yukawa couplings are generated by a dimension five operator, with all fields sitting in a representation not larger than the adjoint. This concerns also the messenger sector of a possible UV completion. Especially the Georgi-Jarlskog factor of -3 [56] is here not possible anymore. There are only three α_{ij} left, which are 1, $-3/2$, and 6, giving five valid combinations as listed in table 4, where we give again typical values for the parameters and the predictions for $\sin\theta_{13}$, including its standard deviation.

$\{\alpha, \beta, \beta', \gamma\}$	$\{a, b, b', \omega\}$	$\sin\theta_{13}$
$\{1, -3/2, -3/2, 6\}$	$\{0.0899, 0.246, 0.679, 0.145\}$	0.114 ± 0.014
$\{1, -3/2, 6, 6\}$	$\{0.0286, 0.212, 0.153, -2.57\}$	0.103 ± 0.008
$\{-3/2, 1, 6, 6\}$	$\{0.0224, 0.217, 0.186, -2.34\}$	0.122 ± 0.015
$\{6, 1, 6, 6\}$	$\{0.0155, 0.281, 0.259, 0.278\}$	0.175 ± 0.009
$\{6, -3/2, 6, 6\}$	$\{0.0134, 0.247, 0.184, -2.77\}$	0.137 ± 0.014

Table 4. Possible CG coefficients with Higgs fields in representations not larger than the adjoint. We also show typical values for the entries of $\hat{\lambda}_D$, where c is normalised to one, and we give the prediction for $\sin\theta_{13}$ inlcuding its 1σ standard deviation.

$\{\beta, \beta', \gamma\}$	$\{b, b'\}$	$\sin\theta_{13}$
$\{-1/2, -3/2, -3\}$	$\{0.217, 0.267\}$	0.094 ± 0.003
$\{-1/2, 3/2, -3\}$	$\{0.216, 0.268\}$	0.094 ± 0.003
$\{-1/2, 6, 6\}$	$\{0.251, 0.240\}$	0.164 ± 0.013
$\{1, -3, 6\}$	$\{0.212, 0.273\}$	0.094 ± 0.003

Table 5. Possible Clebsch Gordan coefficients with a texture zero in the 11 element, $a = 0$. We also show typical values for the entries of $\hat{\lambda}_D$, where c is normalised to one, and we give the prediction for $\sin\theta_{13}$ inlcuding its 1σ standard deviation.

It is interesting to note that this possibility, as well as the last possibility, can be ruled out not only by a precise measurement of the leptonic mixing parameters, but also by a measurement of the SUSY spectrum. A CMSSM like spectrum with a positive μ parameter prefers a ratio m_μ/m_s in the region of 4.5–6 [44, 45], ruling out the special case mentioned before. To get a small ratio $m_\mu/m_s \approx 3$, a spectrum more similar to an AMSB like scenario is preferred, in which the sign of the QCD part of the SUSY threshold corrections is flipped compared to the CMSSM with $\mu > 0$.

3.3 Results for $a = 0$

The next scenario we discuss is a scenario, where we have a texture zero in the 11 element, $a = 0$. This can be motivated by having a flavon vacuum alignment, which has a zero in this position or having a Froggat-Nielsen mechanism at work, which puts there a zero or suppresses this element very strongly. For the CG coefficients we take all the possible values in table 1. In this case we end up with four possible combinations, which are listed in table 5. Note that in this case there are no physical phases.

3.4 Results for symmetric mass matrices

In the (anti-)symmetric case $|\lambda_{12}^D| = |\lambda_{21}^D|$ and $|\lambda_{12}^E| = |\lambda_{21}^E|$, which implies $b = \pm b'$ and $\beta = \beta'$, we find 26 possible combinations listed in table 6. Such a mass matrix is generated, if the 12 and the 21 entries are coming from the same operator. Note that by choosing the unphysical phases appropriately we can always make $b = b'$.

$\{\alpha, \beta, \gamma\}$	$\{a, b, \omega\}$	$\sin\theta_{13}$
$\{-1/2, -3/2, -3\}$	$\{0.122, 0.259, -0.183\}$	0.0903 ± 0.0008
$\{-1/2, 3/2, -3\}$	$\{0.125, 0.255, -0.0985\}$	0.0903 ± 0.0008
$\{-3/2, -3, -3\}$	$\{0.115, 0.233, -0.0736\}$	0.164 ± 0.007
$\{-3, -3/2, -3\}$	$\{0.0205, 0.284, 0.243\}$	0.098 ± 0.002
$\{-3, 3/2, -3\}$	$\{0.0143, 0.268, 0.201\}$	0.098 ± 0.002
$\{6, -3, -3\}$	$\{0.0186, 0.205, -3.08\}$	0.139 ± 0.001
$\{9, -3, -3\}$	$\{0.0142, 0.212, -3.04\}$	0.144 ± 0.003
$\{-18, -3/2, -3\}$	$\{0.0028, 0.257, -0.294\}$	0.0901 ± 0.0008
$\{-18, 3/2, -3\}$	$\{0.0033, 0.255, -0.187\}$	0.0900 ± 0.0007
$\{-18, -3, -3\}$	$\{0.0120, 0.268, -0.0757\}$	0.183 ± 0.004
$\{1, -3, 9/2\}$	$\{0.115, 0.234, 0.195\}$	0.105 ± 0.009
$\{3/2, 9/2, 9/2\}$	$\{0.093, 0.186, 0.107\}$	0.128 ± 0.003
$\{6, -3, 9/2\}$	$\{0.0254, 0.289, 0.108\}$	0.1325 ± 0.0009
$\{9, -3, 9/2\}$	$\{0.0155, 0.275, 0.137\}$	0.129 ± 0.003
$\{-18, -3, 9/2\}$	$\{0.0057, 0.240, 2.97\}$	0.107 ± 0.002
$\{-18, 9/2, 9/2\}$	$\{0.0117, 0.209, -3.05\}$	0.149 ± 0.003
$\{-18, 6, 9/2\}$	$\{0.0183, 0.258, -3.10\}$	0.184 ± 0.002
$\{1, -3, 6\}$	$\{0.127, 0.258, -0.114\}$	0.0903 ± 0.0008
$\{1, 9/2, 6\}$	$\{0.0888, 0.181, 0.200\}$	0.097 ± 0.002
$\{3/2, 9/2, 6\}$	$\{0.111, 0.225, 0.177\}$	0.11 ± 0.01
$\{9/2, -3, 6\}$	$\{0.0200, 0.280, 0.105\}$	0.100 ± 0.001
$\{9/2, 9, 6\}$	$\{0.0918, 0.179, 0.060\}$	0.183 ± 0.002
$\{6, -3, 6\}$	$\{0.0182, 0.280, 0.249\}$	0.098 ± 0.002
$\{9, -3, 6\}$	$\{0.0108, 0.263, 0.286\}$	0.094 ± 0.003
$\{-18, 9/2, 6\}$	$\{0.0094, 0.221, -2.96\}$	0.116 ± 0.002
$\{-18, 6, 6\}$	$\{0.0133, 0.211, -3.08\}$	0.143 ± 0.003

Table 6. Possible Clebsch-Gordan coefficients with a symmetric mass matrix and the resulting prediction for $\sin\theta_{13}$.

This case cannot be combined with any other case. If we restrict ourselves to certain operators or choose $a = 0$, no combination remains viable.

4 Possible implications for Dirac CP violation in the lepton sector

The results of the T2K and MINOS experiments [1, 3] and of the global analysis of the neutrino oscillation data [4] have important implications for the Dirac CP violation in the lepton sector when the unitary mixing matrix $U_\nu^\dagger$, originating from the diagonalisation of the neutrino mass matrix, is (up to CP violating diagonal phase matrices) of BM or TBM form, while that originating from the diagonalisation of the charged lepton mass matrix
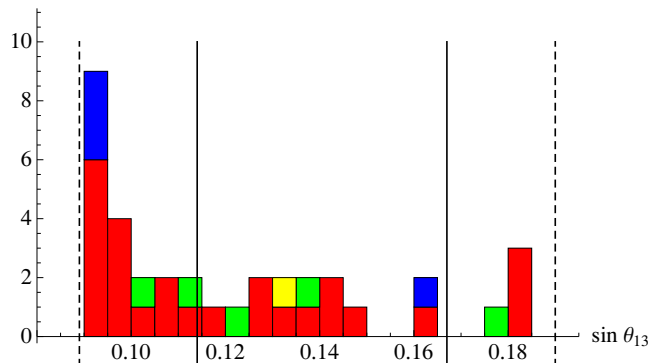
Figure 1. Graphical presentation of the results obtained. The vertical straight (dashed) lines denote the 1σ (2σ) allowed ranges of $\sin\theta_{13}$ taken from [4] (see also table 7). The yellow, green, blue, red bins correspond to the results from table 3, 4, 5, 6.

can be approximated as $U_e = R_{12}(\theta_{12}^e)$ (see eq. (2.2)). We recall that if $U_\nu^\dagger$ coincides with the BM mixing matrix one has:

$$\sin^2\theta_{12}^\nu = \frac{1}{2}, \qquad \sin^2\theta_{23}^\nu = \frac{1}{2}, \qquad \sin^2\theta_{13}^\nu = 0. \tag{4.1}$$

In the case of tri-bimaximal mixing form of $U_\nu^\dagger$,

$$\sin^2\theta_{12}^\nu = \frac{1}{3}, \qquad \sin^2\theta_{23}^\nu = \frac{1}{2}, \qquad \sin^2\theta_{13}^\nu = 0. \tag{4.2}$$

While in both cases θ_{23}^ν coincides, or is close to, the experimentally determined best fit value of the atmospheric neutrino mixing angle θ_{23}, in the case of bimaximal mixing form of $U_\nu^\dagger$ a relatively large correction to θ_{12}^ν is needed to get a value of the solar neutrino mixing angle θ_{12} compatible with that determined from the data. And in both cases a non-zero θ_{13}, having a value in the range $\sin^2\theta_{13} = 0.021\,(0.025)\pm 0.007$, see also eq. (1.1), suggested by the current data, has to be generated.

If $U_\nu^\dagger$ has bimaximal form, the angle $\theta_{12}^\nu = \pi/4$ is corrected by the charged lepton mixing as follows:

$$\sin^2\theta_{12} \simeq \frac{1}{2}\frac{1 + \frac{1}{\sqrt{2}}\cos\phi\sin 2\theta_{12}^e - \frac{1}{2}\sin^2\theta_{12}^e}{1 - \frac{1}{2}\sin^2\theta_{12}^e}, \quad \text{(BM)}, \tag{4.3}$$

where we have neglected the possible contributions from the charged lepton mixing angles θ_{13}^e and θ_{23}^e.[4] To leading order in $\sin\theta_{12}^e$ we get [28–31, 60–64]:

$$\sin^2\theta_{12} \simeq \frac{1}{2} + \frac{1}{\sqrt{2}}\cos\phi\,\sin\theta_{12}^e \simeq \frac{1}{2} + \cos\delta\,\sin\theta_{13}, \quad \text{(BM)}, \tag{4.4}$$

where we have used the relation $\cos\phi = \cos\delta$ and the fact that in the approximation employed we have:

$$\sin\theta_{13} \simeq \frac{1}{\sqrt{2}}\sin\theta_{12}^e > 0. \tag{4.5}$$

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[4]These contributions are given in [28, 29, 31].

	$\sin\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$
Best fit	0.145	0.306	0.42
1σ	0.114–0.167	0.291–0.324	0.39–0.50
2σ	0.089–0.190	0.275–0.342	0.36–0.60
3σ	0.032–0.210	0.259–0.359	0.34–0.64

Table 7. Results of the global fit of the PMNS mixing angles taken from [4] and used in our analysis. The results quoted were obtained using the "old" reactor $\bar{\nu}_e$ fluxes (see [4] for details).

The sign of the second term in the r.h.s. of the equations in (4.4) is important[5] in view of the fact that, according to [4], the global neutrino oscillation data favors a negative value of $\cos\delta\sin\theta_{13}$ over the positive value. We note that such a sign is in agreement with the one needed to reduce the bimaximal prediction for θ_{12} down to the experimentally allowed range.

In the case of the TBM form of $U_\nu^\dagger$ we find under the same assumptions:

$$\sin^2\theta_{12} \simeq \frac{1}{3}\frac{1+\cos\phi\sin 2\theta_{12}^e}{1-\frac{1}{2}\sin^2\theta_{12}^e}\,, \quad \text{(TBM)}. \tag{4.6}$$

To leading order in $\sin\theta_{12}^e$ we get:

$$\sin^2\theta_{12} \simeq \frac{1}{3} + \frac{2\sqrt{2}}{3}\cos\delta\,\sin\theta_{13}\,, \quad \text{(TBM)}, \tag{4.7}$$

where we have used again $\cos\phi = \cos\delta$ and eq. (4.5), which is valid also in this case.

The angle $\theta_{23}^\nu = \pi/4$ gets the same correction in both cases of BM and TBM $U_\nu^\dagger$:

$$\sin^2\theta_{23} \simeq \frac{1}{2}\frac{\cos^2\theta_{12}^e}{1-\frac{1}{2}\sin^2\theta_{12}^e}\,. \tag{4.8}$$

As it follows from the above expression, the leading correction is of order $\sin^2\theta_{12}^e$ [28, 29].

The values of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$, obtained in the global data analysis in [4] are shown for convenience in table 7. In figure 2 we present graphically the constraints on $\cos\delta$ implied by the data on $\sin^2\theta_{12}$ and $\sin\theta_{13}$ in both cases of BM and TBM $U_\nu^\dagger$, using the relations (4.3), (4.6), which are exact in θ_{12}^e.

A few comments are in order. It follows from our analysis that in the case of bimaximal $U_\nu^\dagger$, the current 3σ (2σ) experimentally allowed range for $\sin^2\theta_{12}$ requires that $\sin\theta_{13} \gtrsim 0.14$ (0.16), the minimal value being very close to the best fit value found in [4]. If future data on θ_{13} will show (taking into account all relevant uncertainties) that, e.g. $\sin\theta_{13} \lesssim 0.10$, the simple case under discussion of $U_\nu^\dagger$ having a BM mixing form and $U_e = R_{12}(\theta_{12}^e)$ will be ruled out. Further, using the 3σ (2σ) allowed ranges of both $\sin^2\theta_{12}$ and $\sin\theta_{13}$ we find that $\cos\delta$ is constrained to lie in the interval: $-1 \leq \cos\delta \lesssim -0.60$ (-0.79). Thus,

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[5]We note that in [28, 29, 31, 60–64] a somewhat different parametrisation of the PMNS matrix was used (namely, $U = U_e^\dagger U_\nu$ and $U = R_{12}^T(\theta_{12}^e)\Phi R_{23}(\hat{\theta}_{23})R_{12}(\hat{\theta}_{12})Q$) and instead of the first relation in (4.4), the relation $\sin^2\theta_{12} \simeq 1/2 - (\cos\phi\sin\theta_{12}^e)/\sqrt{2}$ was obtained. It is not difficult to show that in this case we have $\cos\delta = -\cos\phi$ and thus one arrives at the same result for the relation between $\sin^2\theta_{12}$, θ_{13} and δ.
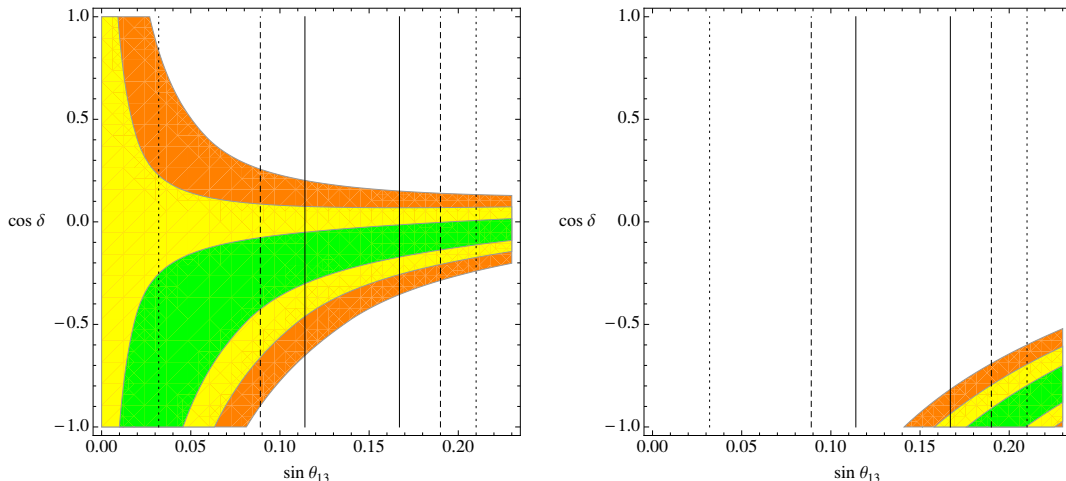
Figure 2. The cosine of the Dirac CP phase δ as a function of $\sin\theta_{13}$ in the cases of tri-bimaximal (left panel) and bimaximal mixing (right panel) arising from the diagonalisation of the neutrino mass matrix, eqs. (4.3) and (4.6). The green, yellow, orange regions correspond to the 1, 2, 3σ allowed ranges of $\sin^2\theta_{12}$. In the bimaximal mixing case only the negative values of $\cos\delta$ are compatible with the data on $\sin^2\theta_{12}$. The vertical straight, dashed, dotted lines denote the 1, 2, 3σ allowed ranges of $\sin\theta_{13}$. The values of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ are taken from [4] (see text for details).

$\cos\delta = 0$, or $\delta = \pi/2$, and therefore maximal CP violation in neutrino oscillations, is ruled out in the scheme we are considering.

We get very different constraints on $\cos\delta$ in the case of the TBM form of $U_\nu^\dagger$. If we use the 3σ allowed ranges of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ in the analysis, all possible values of $\cos\delta$ are allowed. The 2σ intervals of allowed values of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ require that $-0.66 \lesssim \cos\delta \lesssim 0.09$. Thus, maximal CP violation, $\delta = \pi/2$, is allowed.

Using the constraints on δ derived above, we have obtained also predictions for the magnitude of the rephasing invariant J_{CP}. The latter has the following form to leading order in $\sin\theta_{12}^e$ in the cases of BM and TBM $U_\nu^\dagger$:

$$J_{\mathrm{CP}}^{\mathrm{BM}} \simeq \frac{1}{4}\sin\delta\sin\theta_{13}\,, \tag{4.9}$$

$$J_{\mathrm{CP}}^{\mathrm{TBM}} \simeq \frac{1}{3\sqrt{2}}\sin\delta\sin\theta_{13}\,. \tag{4.10}$$

The results of this analysis are presented in figure 3. As could be expected on the basis of the results obtained for $\cos\delta$, the existing data on $\sin^2\theta_{12}$ and $\sin\theta_{13}$ imply that in the case of BM form of $U_\nu^\dagger$, J_{CP} should lie in the interval $|J_{\mathrm{CP}}| \lesssim 0.037(0.031)$, using the 3$\sigma$ (2σ) ranges, the value $J_{\mathrm{CP}} = 0$ being allowed. We get similar results for the maximal possible value of $|J_{\mathrm{CP}}|$ if $U_\nu^\dagger$ exhibits TBM mixing form. However, if we use the 2σ allowed ranges of $\sin^2\theta_{12}$ and $\sin\theta_{13}$, we find that $|J_{\mathrm{CP}}|$ has to be non-zero and not smaller than 0.014: $0.014 \lesssim |J_{\mathrm{CP}}| \lesssim 0.037$. Values of $|J_{\mathrm{CP}}| \gtrsim 0.01$ are potentially accessible by the accelerator experiments T2K, NOνA, etc. planning to search for CP violation effects in neutrino oscillations (see, e.g. [65]).
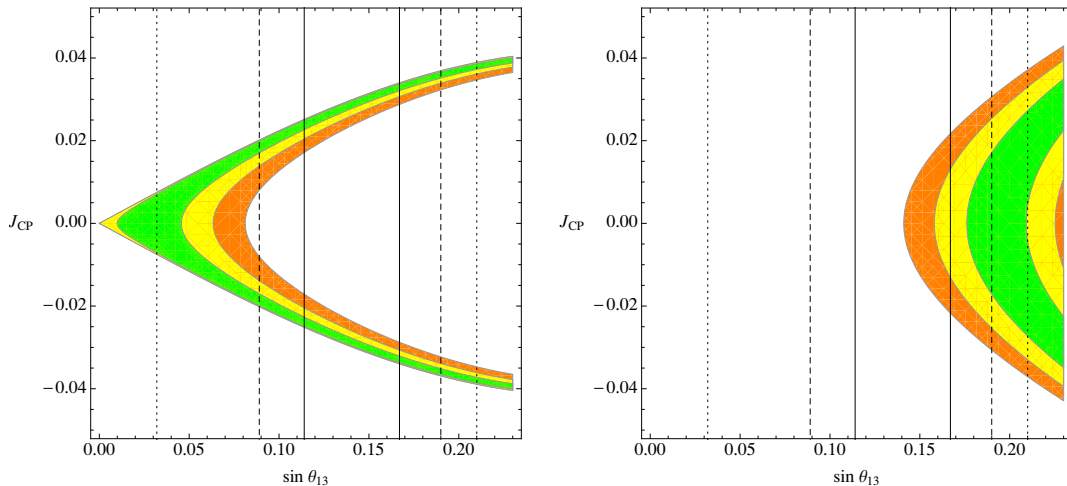
Figure 3. The rephasing invariant $J_{\rm CP}$ as a function of $\sin\theta_{13}$ in the cases of tri-bimaximal (left) and bi-maximal (right) mixing from the neutrino sector. The green, yellow, orange regions correspond to the 1, 2, 3σ allowed ranges of $\sin^2\theta_{12}$. The vertical straight, dashed, dotted lines show the 1, 2, 3σ allowed ranges of $\sin\theta_{13}$. The 1, 2, 3σ allowed ranges of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ are the same as in figure 2.

5 Summary and conclusions

In the present article we have considered the possibility that a sizeable value of the neutrino mixing angle θ_{13}, compatible with the recent indications from the T2K, MINOS and the global neutrino oscillation data, arises primarily from the contribution of the charged lepton sector U_e to the lepton mixing, $U = U_e\, U_\nu^\dagger$, where U is the PMNS matrix and $U_\nu^\dagger$ arises from the diagonalisation of the neutrino mass matrix. This scenario should necessarily be taking place in the context of a wide variety of models predicting a relatively small contribution to θ_{13} from the neutrino sector, say $\theta_{13} \lesssim \mathcal{O}(\lambda^2)$, where λ is the Cabibbo expansion parameter $\lambda \approx 0.23$. The analysis has been performed in the context of SU(5) supersymmetric models, which are independently well motivated and allow the charged lepton contribution to θ_{13} to be related to the mass and mixing observables in the quark sector.

The naive relation $m_E = m_D^T$ between the charged lepton and down quark mass matrices is not compatible with the indication of a relatively large θ_{13}. This is not surprising, as the quoted relation is not compatible with the well known values of the charged fermion mass ratios either and it has to be corrected, possibly by Clebsch-Gordan (CG) factors. This is particularly true in the case (relevant for us) of the charged fermions belonging to the first two families, whose small masses can arise from non-renormalizable operators that can well involve SU(5) breaking sources. Under motivated hypotheses for the values of such CG coefficients, we found all the combinations that are experimentally viable. The main results are summarized in tables 3, 4, 5, and 6. In order to describe the present data, one needs either a non-vanishing 11 entry in the down quark and charged lepton mass matrices, or CG coefficients from non-renormalizable or from SU(5) breaking operators in representations larger than the adjoint.

We also studied the specific case in which the contribution from the neutrino sector to the lepton mixing, $U_\nu^\dagger$ in eq. (2.1), is, up to the phase matrices Φ and Q, see, eq. (2.2), of the bimaximal (BM) or tri-bimaximal (TBM) form, while that from the charged lepton sector is assumed to be of the form $U_e = R_{12}(\theta_{12}^e)$. Under these assumptions, $\Phi = \mathrm{diag}(1, e^{i\phi}, 1)$, ϕ being a CP violating phase, and Q contains the two Majorana CP violating phases. In this case, it is possible to draw important conclusions on the leptonic (Dirac) CP-violating phase $\delta \approx -\phi$ and on the magnitude of the CP violation effects in neutrino oscillations. The latter, as is well known, is determined by the rephasing invariant J_{CP}. Our results are summarized in figures 2, 3. We find that in the case of bimaximal $U_\nu^\dagger$, the current 3σ (2σ) experimentally allowed range for $\sin^2\theta_{12}$ requires that $\sin\theta_{13} \gtrsim 0.14$ (0.16). If future data on θ_{13} will show (taking into account all relevant uncertainties) that $\sin\theta_{13}$ has a smaller value, e.g. $\sin\theta_{13} \lesssim 0.10$, the simple case of U_ν having a BM mixing form and $U_e = R_{12}(\theta_{12}^e)$ will be ruled out. Further, using the 3σ (2σ) allowed ranges of both $\sin^2\theta_{12}$ and $\sin\theta_{13}$ we find that $\cos\delta$ is constrained to lie in the interval: $-1 \leq \cos\delta \lesssim -0.60$ (-0.79). Thus, $\cos\delta = 0$, or $\delta = \pi/2, 3\pi/2$, and therefore maximal CP violation in neutrino oscillations, is essentially ruled out in the scheme we have considered. If it will be confirmed experimentally that $\cos\delta \sin\theta_{13} < 0$, that would imply (in the standard parameterization of the PMNS matrix, in which $\sin\theta_{13} > 0$) that $127°$ ($142°$) $\lesssim \delta \lesssim 233°$ ($218°$). Correspondingly, J_{CP} should lie in the 3σ (2σ) interval $|J_{\mathrm{CP}}| \lesssim 0.037$ (0.031), the value $J_{\mathrm{CP}} = 0$ being allowed. However, if it will be experimentally established that $\cos\delta \sin\theta_{13} > 0$, that would rule out the case of $U_e = R_{12}(\theta_{12}^e)$ and $U_\nu^\dagger$ having a BM mixing form.

In the case of the TBM form of $U_\nu^\dagger$ and $U_e = R_{12}(\theta_{12}^e)$, all possible values of $\cos\delta$ are allowed if one uses the 3σ allowed ranges of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ in the analysis. The 2σ intervals of allowed values of $\sin^2\theta_{12}$ and $\sin\theta_{13}$ require that $-0.66 \lesssim \cos\delta \lesssim 0.09$, or $85° \lesssim \delta \lesssim 131°$. Thus, maximal CP violation, $\delta = \pi/2$, in both cases is allowed. We find using 2σ allowed ranges of $\sin^2\theta_{12}$ and $\sin\theta_{13}$, that $|J_{\mathrm{CP}}|$ has to be non-zero and not smaller than 0.014: $0.014 \lesssim |J_{\mathrm{CP}}| \lesssim 0.037$. This interval falls in the range of the potential sensitivity of the T2K and the future neutrino neutrino oscillation experiments (NOνA, etc.), designed to search for CP violation effects in neutrino oscillations.

Acknowledgments

Note added. During the finalising stage of the work on the present article, ref. [66] appeared, where the authors follow a similar approach in generating a relatively large θ_{13}. Our work goes beyond the analysis performed in [66] in a number of aspects. In [66] only the case $a = 0$ is discussed, while we consider the more general case of $a \neq 0$. The authors of [66] use only leading order terms (without error estimates) in the expressions

for the observables in terms of the parameters of the down quark and charged lepton mass matrices (i.e., in eqs. (12a)–(12e)) and they do not utilise the rather precisely known value of the ratio m_s/m_d (see table 2). This rules out some of the possibilities considered in [66] to be viable. Finally, the authors of [66] draw only qualitatively conclusions about the value Dirac CP violating phase δ in the case of (tri-)bimaximal mixing arising from the neutrino sector. We perform a quantitative analysis and determine the allowed ranges (at 1, 2 and 3σ) of values of $\cos\delta$ and of the rephasing invariant $J_{\rm CP}$ in the two cases (figures 2 and 3).

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