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FOR ADVANCED STUDIES

Environmental Effects on the Luminositiy-Diameter Relation for Disk Galaxies

Thesis submitted for the degree of "Magister Philosophiae"

Astrophysics Sector

Candidate:
Marisa Girardi

Supervisor:
Prof. D.W. Sciama

October 1990

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Introduction

For many years, astronomers have known that for a given sample of galaxies the frequency distribution of Hubble types depends on the environment from which the sample is selected: elliptical and S0 galaxies are the dominant population in the densest region of rich clusters, whereas spirals are found most frequently in settings of much lower density. However this is not the only influence of the environment, in chapter 1 there is a short description of the galaxy properties depending on environment: in particular, I will limit my work to disk galaxies, that is from lenticulars to irregulars. One of the best defined empirical relationship in astronomy is the luminosity-diameter relation (described in chapter 2), that links the two most fundamental photometric properties of galaxies. Moreover, this relation is tight enough to search for possible dependence on environment. So, several authors looked for possible differences in luminosity-diameter relation for galaxies in different settings: from rich cluster galaxies to field galaxies. On the other hand, some theorical mechanisms exist that may explain possible effects on galaxy structures (see chapter 3). Nevertheless, actually there is no general consent on the dependence of this relation on environment. A very recent paper on elliptical galaxies (Giuricin et al. 1989) and controversial results about disk galaxies induced me to reconsider the environmental dependence of the luminosity-diameter relation for disk galaxies (see chapter 4).

Chapter 1

Environment and Disk Galaxies

Summary

It is well known that the frequency distribution of galaxy morphological types depends on the environment, since ellipticals and lenticulars dominate the dense environments, such as the core of rich clusters, and spirals dominate the field. Several other correlations between galaxy properties and their environment are studied in the literature (in my thesis, I discuss disk galaxies). There are some evidences that the disk galaxies in dense environments are HI-deficient, redder and have a reduced star formation rate respect to the galaxies in environments of lower density. Moreover, it is on debate whether the disk galaxies in densest environments have rotation curves falling (and not flat or rising as the galaxies in less dense settings) as if these galaxies have a reduced halo. Also the luminosity (at fixed morphological type) may depend on environment: the more luminous galaxies seem to prefer the densest environments. The galaxy structure may depends on environment too: see, e.g., cD galaxies and those galaxies which present a cut-off in their luminosity profiles.

All the previous considerations induce us to think that environment may strongly affects the galaxy properties. A very suitable tool to study the influence of the environment on galaxies seems to be the relation between galaxy intrinsic luminosity and absolute diameter (the L-D relation), since this relation is very tight.

1.1 Disk Galaxies

For the purposes of the following discussion, we assume that a disk galaxy is composed of two parts: a spheroidal bulge and a flattened disk. It is generally believed, although not well established, that the bulge components of disk galaxies are similar to E galaxies. Disk components, on the other hand, may exhibit a variety of appearances. Some are forming stars at the present epoch, either chaotically (irregular systems) or in a relatively regular pattern (spirals); others appear to have substantially ceased star formation either temporarily or permanently and are characterized by relatively smooth disks (lenticulars or S0 systems). The relative size and luminosity of the bulge and disk components appear to vary continuously from dominated bulge systems (E galaxies) to dominated disk systems (late-type spirals and irregulars). Several classification systems were presented in the literature, the most classical being the Hubble classification (see Mihalas and Binney 1981 for a review): disk galaxies occupy the middle-right part of Hubble's tuning-fork diagram, and correspond to types from -3 to 10 in the revised de Vaucouleurs system. A useful first-cut description of a disk galaxy might be provided by: i) the bulge-disk ratio (B/D) of the system, ii) the presence or absence of current-epoch star formation in the disk, and iii) the spatial regularity of the current epoch star-forming episodes (arms).

The light distribution of the bulge appears, at first glance, to be well fitted by a de Vaucouleurs' law (e.g. Kormendy 1977a), although more careful study suggests that this may not be the case (see e.g. Spinrad et al 1978). The light distribution in the disk appears to follow an exponential law, although the range of applicability of such a simple description has been questioned (e.g. Kormendy 1977a). In particular, the exponential law that represents the disk light distribution in the outer parts may not be applicable in the bulge region; in fact, the contribution of the disk component in the central regions of some galaxies may be negligible. Other details on the light distribution for disk galaxies are presented in section 2.1.

In the following sections I describe the principal effects of the environment on disk galaxies. Also some hints about ellipticals are reported.

1.2 Frequency Distribution in Different environments

E and S0 galaxies dominate the relatively regular and dense and spiral-poor clusters, while spirals dominate lower-density irregular clusters and the field. Hubble and Humason (1931), Morgan (1961), and Abell (1965) described the transition to earlier-type galaxies (Es and S0s) in rich clusters, and Oemler (1974) quantified the relationship by identifying characteristic global mixes of E, S0, and spiral galaxies. Melnick and Sargent (1977) and Bahcall (1977) studied the distribution of actively star-forming disk systems (spirals and irregulars) and non-star-forming disk systems (S0's) in several rich, dense clusters known to be X-ray emitters. The ratio N(S)/N(S0) increases monotonically with increasing distance from the center of the clusters in their sample. Dressler (1979) suggested that this systematic behavior is representative of a more general correlation between N(S)/N(S0) and local galaxy density. He also found that the luminosity of the bulge component of disk galaxies depends on local galaxy density, the bulge luminosity tends to be higher in denser environments. Dressler (1980) confirmed the previous result and showed that there is a tight relation between the galaxy morphology and the local galaxy density (see Fig. 1.1), and this behavior has been shown to extend all the way to the low-density field (Bhavsar 1981, de Souza et al. 1982, Postman and Geller 1984, Giuricin et al. 1985a), about five orders of magnitude in number density. The fraction of spiral galaxies decreases as the fraction of S0s and Es increases with local galaxy density, almost independently of global cluster characteristics. This "morphology-density" relation is monotonic but its variation is extremely slow (roughly logarithmic), so that the low-density field is dominated by 80-90% spirals and the highest-density regions contain 80-90% elliptical and S0 galaxies. All types are represented in all environments. However, Salvador-Sole', Sanroma', and Jordana (1989) have suggested that the global properties of a cluster may also be important in determining the morphology of galaxies in clusters and perhaps the correlation with global properties may actually be the fundamental correlation (see also Whitmore, 1989). In Fig. 1.2 the morphology content versus cluster center distance is represented.

The galaxies of compact groups follow a qualitatively similar morphology-density relation, which is however offset to higher densities (the fraction of

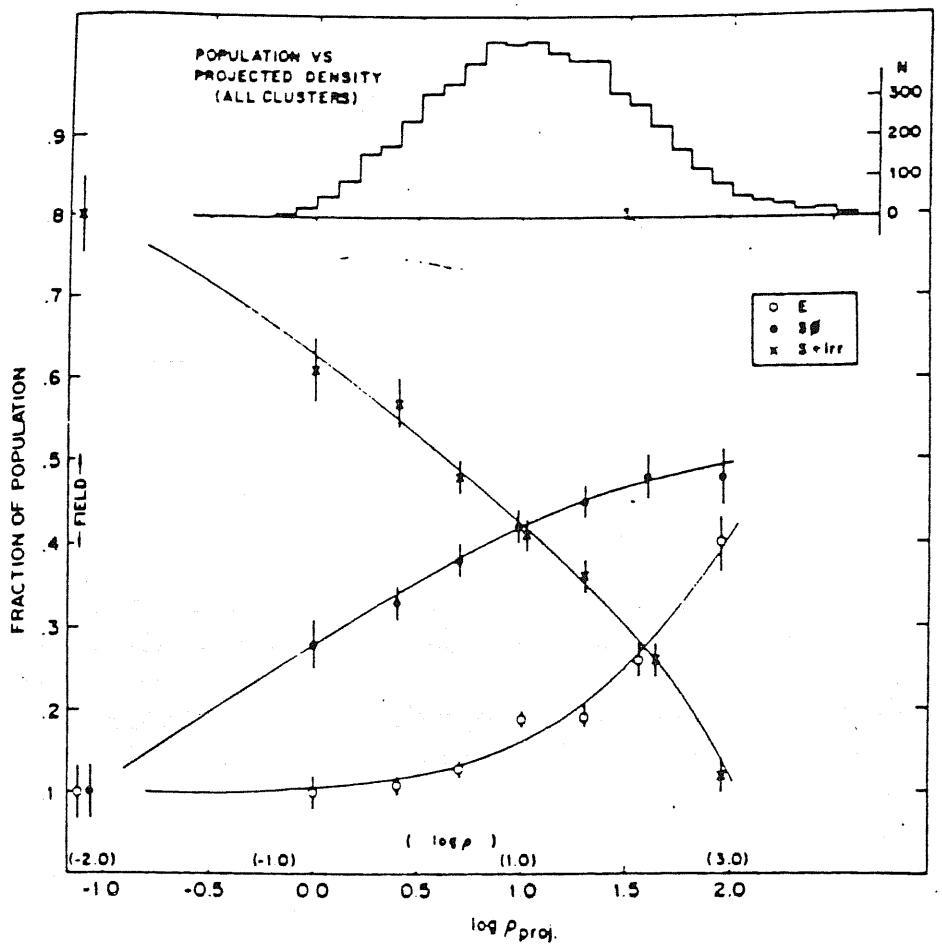

Figure 1.1: The fraction of E, S0, and S+I galaxies as a function of the log of the local projected density, in galaxies Mpc^{-3} for 55 clusters. An estimated scale of true space density in galaxies Mpc^{-3} is also included. The upper histogram shows the number distribution. (from Dressler, 1980)

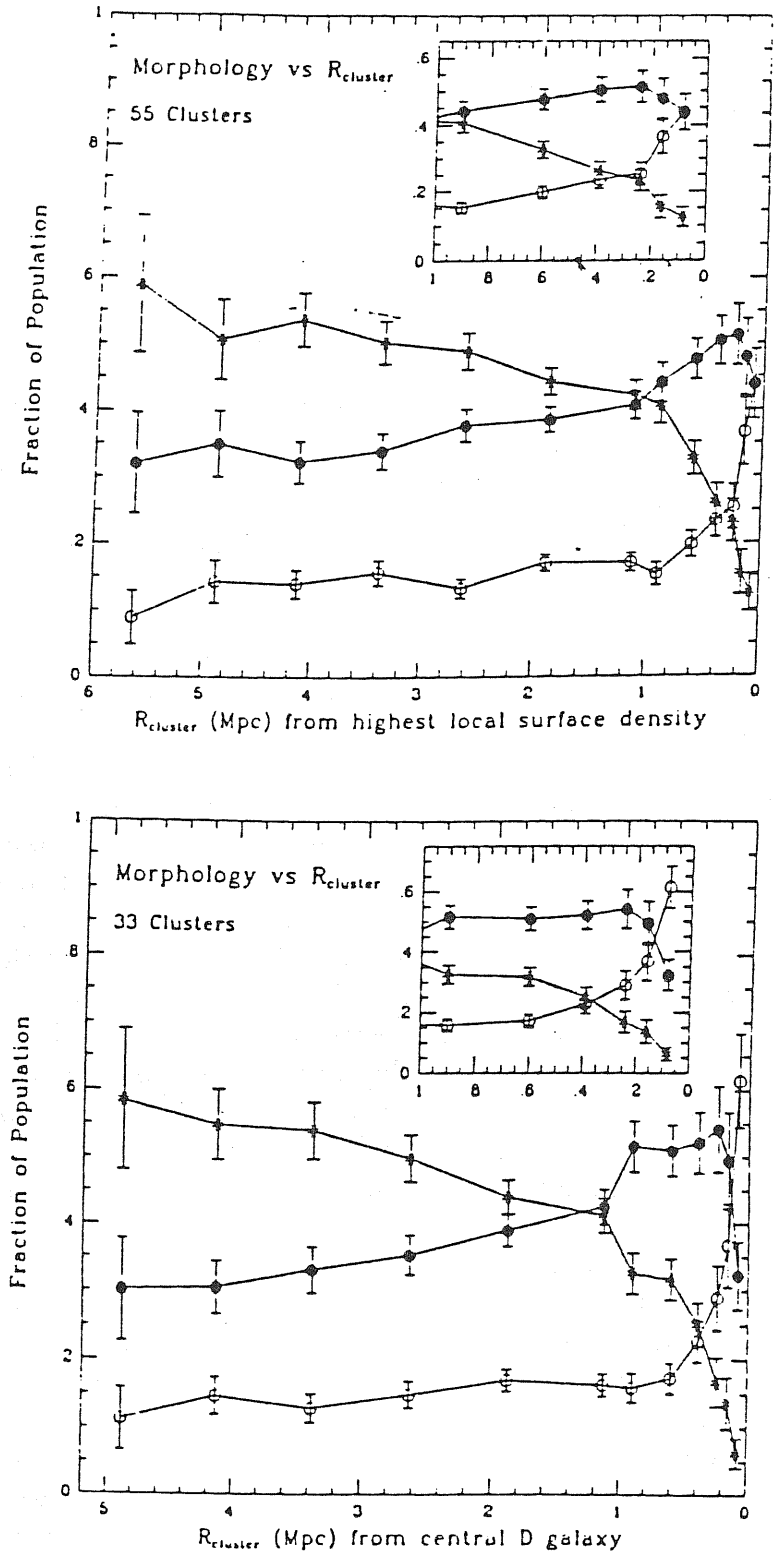

Figure 1.2: a) The fraction of E (open circles), S0 (filled circles), and S+I (asterisks) galaxies as a function of the distance from the center of the cluster, $R_{cluster}$. The upper insert shows a blow up of the central 1Mpc. b) Same as a) but only including the 33 clusters containing D galaxies. (from Whitmore and Gilmore, 1990)

early-type galaxies is considerably lower in compact groups than in cluster regions of comparable densities, see Hickson et al. 1988). Some authors have also suggested a dependence of the morphology-density relation in groups on the velocity dispersion (Hickson et al., 1988) and on the morphology of the first-ranked galaxy (Ramella et al., 1987).

1.3 Orientation and Alignment

If galaxies formed by fragmentation of larger structures, according to the dissipative pancake scenario, one expects some sort of correlation between the orientation of a galaxy and the orientation of its parent pancake (see e.g. Doroshkevich, 1970). However, there is a long controversy associated with the search for anisotropy in the orientations of galaxies. There is a list of clusters in which galaxy alignment has been noted (e.g. Thompson 1976, Dressler 1976) for disk systems, or ellipticals, or both. Adams et al. (1979) also suggested that the major axes of disk galaxies are aligned in two preferred directions: either along or perpedicular to the clusters major axis. Similarly, supercluster environment is studied. The north pole of the Milky Way indeeed lies in the plane of the Local Supercluster, but most searches for statistically significant alignments of such sort in the Local Supercluster came up with null results (Helou and Salpeter 1982; McGillivray et al. 1982; Kaprandis and Sullivan 1983; Flin and Godlowski 1984), except for small-scale antialignment of galaxies in binaries (Helou 1984). Flin and Godlowski (1985, 1986), however, claim to find an alignment when face-on galaxies are included; the galactic minor axes tend to lie parallel to the supergalactic plane, and actually to point toward the Virgo Cluster. Perhaps the strongest indication for galaxy alignment has been reported in the Perseus supercluster (Gregory, Thompson, and Tifft 1981), where the position angles of the 141 galaxies studied tend to lie either along the major axis of the supercluster or in a direction roughly 60° away from it. Flin (1988) found that planes of galaxies belonging to the Perseus Supercluster are perpendicular to the Local Supercluster plane, but the main contribution to the anisotropy is due to elliptical galaxies because the spirals show very weak alignment (see Fig. 1.3). Flin (1990) confirmed the same alignment of galaxies in Local and Perseus Superclusters. On the contrary, Godlowski (1990) found that, even if the galaxy planes are mainly perpendicular to the plane of the Local

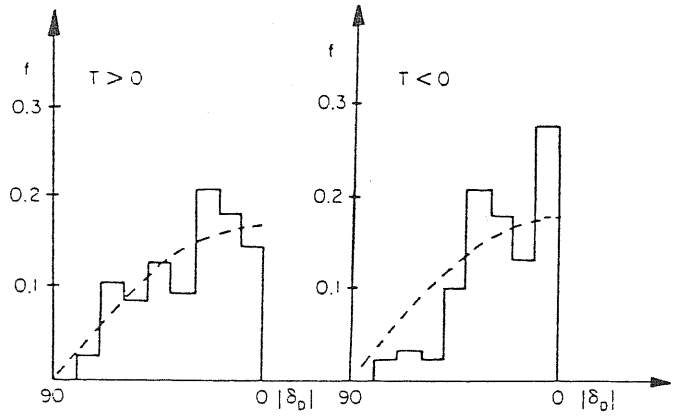

Figure 1.3: The distribution of galaxy rotation axes for different galaxy morphological types, δ_D is the angle between the direction of the galaxy rotation axis and the supercluster plane (broken line illustrates the distribution expected if isotropic). (from Flin, 1988)

Supercluster, in some regions galaxy planes are parallel to it.

1.4 Interstellar Gas Content

As regards the neutral hydrogen in disk system, it is interesting the presence of the "anemic" spirals (van den Bergh 1976) in clusters (Strom and Strom 1979a, Wilkerson 1980, Sullivan et al. 1981). These observations show a tendency for spirals in the intermediate-density environments (there are very few spirals in the densest environments) to be gas poor by factors of 2-3 relative to their field counterparts of the same Hubble type; S0 show a similar tendency (Krumm and Salpeter, 1979a,b). The data have been compiled from HI measurements to discover HI deficiency (Bothun et al. 1982, Giovanelli et al. 1981, Bothun et al. 1982, Haynes, Giovanelli, and Chincarini 1984, Dressler 1986); Warmels (1985) discussed also the presence of truncated and asymmetric HI disks. Other data come from optical measurements of integrated Hβ flux (Kennicutt 1983). In an evolved cluster, as Coma cluster is, all the disk galaxies near the cluster center show a very low quantity of gas content

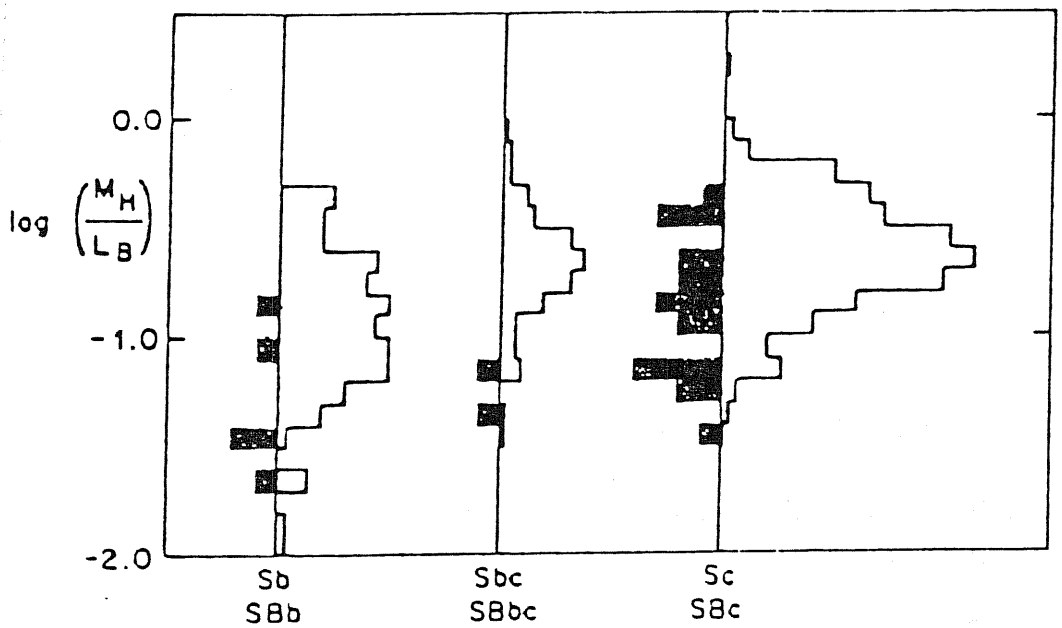

Figure 1.4: A comparison of the distance-independent hydrogen mass to blue luminosity ratio distributions in the Virgo cluster and in the field. For each galaxy type, the left dark-hatched distribution is for the Virgo cluster core, while the right light-hatched distribution is for field spirals. Horizontal scales are not the same. (from Kennicutt, 1983)

(Sullivan et al. 1981). Most of inherent works have been clearly devoted to the galaxies in the Virgo cluster (Davies and Lewis 1973; Huchtmeier, Tamman, and Wendker 1976; Balkowsky, and Gerard 1980; Hoffman et al. 1989). As regards the Virgo galaxies, in the innermost region the galaxies are HI deficient by a factor of 2 to 5, and occasionally, up to 10; moreover bright galaxies seem to be more deficient than faint ones (Chamaraux et al. 1980). Similarly, early type galaxies are far more deficient than late type galaxies (Huchtmeier and Richter, 1989). There is still disagreement over the origin of the gas deficiency. However several theoretical mechanisms exist which are efficient enough to remove gas from a spiral in rich clusters: ram-pressure ablation models (Gunn and Gott, 1972; Lea and de Young, 1976; Gisler, 1978,1979), thermal-evaporation models (Cowie and Songalia, 1977; Balbus and McKee, 1982). Larson et al. (1980) have proposed a mechanism of tidal stripping. It is also possible that the observed, present-day HI deficiency in cluster spirals is the result of an initial defect and that cluster galaxies are simply endowed with less post-formation residual gas than field galaxies.

Searches for [O II] emission were conducted in S0 galaxies located in a variety of environments. The frequency of [O II] emission is lower for the S0's located in rich clusters than in those in the field (Gisler 1978).

CO studies of galaxies (Young and Scoville, 1982; Young, Scoville and Brady, 1985) also provide information on the effect of environment on the dense component of the ISM: the shapes of the CO distributions with radius in the Virgo cluster and the total CO luminosities agree well with those found in field galaxies, suggesting that the molecular contents of these galaxies are not significantly different from that of field galaxies of the same type and luminosity.

1.5 Colours

As regards the colours of galaxies, the fact that spirals in Virgo are redder than their field counterparts has been known since the work by Holmberg (1958). But, from a sample of over 400 galaxies, Visvnathan and Sandage (1977) argued that the colours of S0 galaxies are independent of environment. It should be noted that their result applied to the combined light of the bulge and disk. On the other hand, Strom and Strom (1978e) found the disk colours of S0 galaxies in the outer parts of the Coma cluster to be

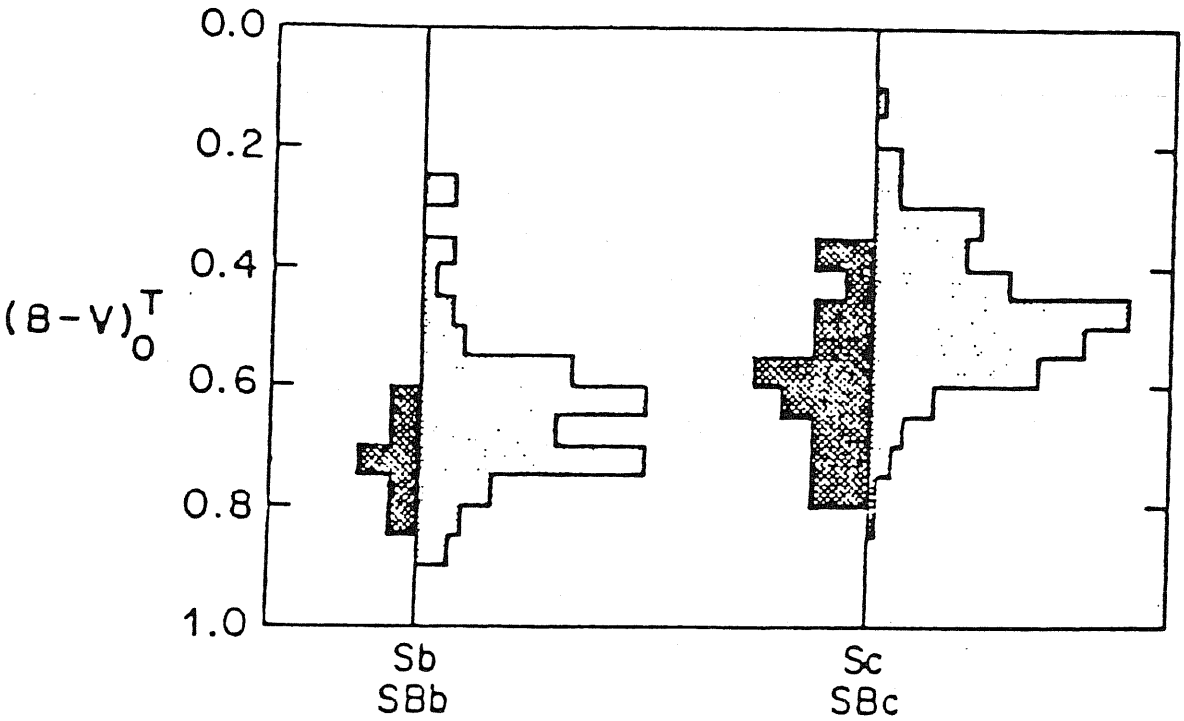

Figure 1.5: Comparison of the (B-V) color distributions in the Virgo cluster and the field. Notation is the same as in Fig.1.4, Virgo to the left, field to the right. (from Kennicutt, 1983)

bluer than the disk colours of those galaxies located in dense central regions. According to Kennicutt (1983) no reasonable combination of selection effects and systematic errors can produce the difference between cluster and field galaxies: the Virgo Sb and Sc galaxies are 0.1 mag redder on average than field galaxies (see Fig. 1.5). Stauffer (1983), however, first pointed out that the colours of the Virgo spirals are normal relative to field galaxies with the same HI content. The HI-colour correlation diagram for field and Virgo galaxies is shown in Fig. 1.6 (Stauffer, 1983). A simple visual comparison of the two figures leads to the following conclusion: i) there is a significant population of quite HI deficient galaxies in Virgo that fall in a region of the diagram that is nearly unoccupied by the field sample, moreover these deficient Virgo galaxies are also quite red; ii) blue, HI rich galaxies, present in the field sample, are absent in Virgo cluster. The most likely explanation is that a deficiency of gas would depress the star formation rate, and hence

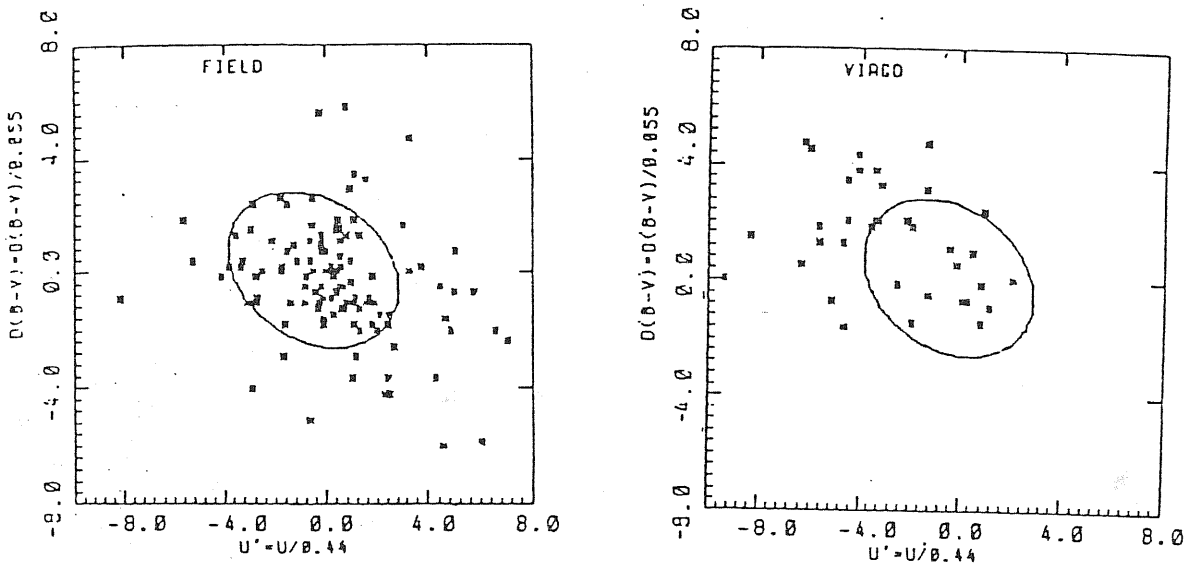

Figure 1.6: HI-color diagram for field and Virgo galaxies. The two axes measures deviations of a galaxy's integral color and HI content from the mean value for that morphological type. Positive $\delta(B-V)$ corresponds to being redder than normal, and positive u' corresponds to being HI rich. (from Stauffer, 1983)

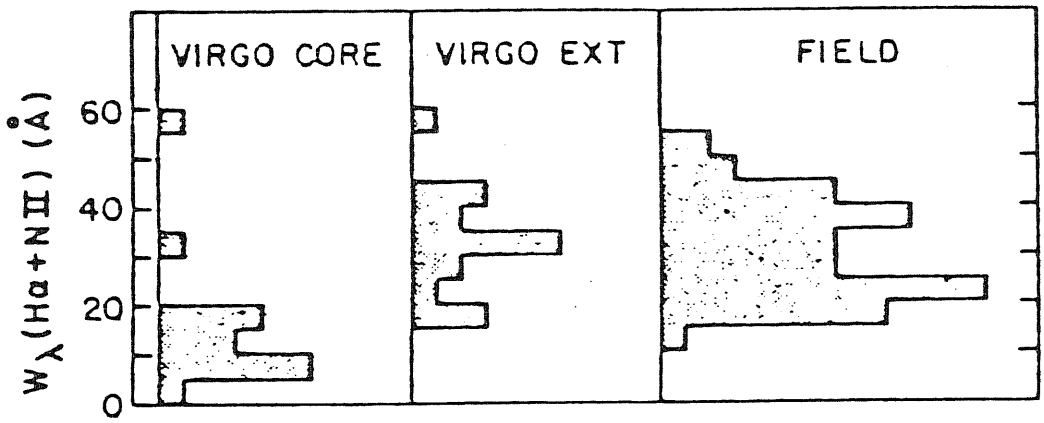

Figure 1.7: Distribution of emission equivalent width among Sc and SBc galaxies, for the Virgo core and the field. The "Virgo Ext" sample is composed of field spirals near, but not within the Virgo cluster. The horizontal scale is different in the field plot. (from Kennicutt, 1983)

the Hα emission, and would in turn gradually act to redder the integrated colours of the galactic disks (Searle, Sargent, Bagnuolo 1973).

1.6 Hα, Far Infrared, Radio Continuum Emissions and Star Formation

The main results are outlined. Hα emission, which is related to OB stars in the galaxy, is an indicator of star formation (see e.g. Chiosi and Renzini, 1986). Kennicutt (1983) found that the emission-line equivalent width Hα+[NII] is weaker in Virgo cluster than in the field (Fig. 1.7). This result suggests that the Virgo environment has significantly altered the gaseous and stellar contents of disks.

Far infrared (FIR) emission from galaxies is generally interpreted as due to radiation by dust heated by the stellar photon field, and there are some evidences that FIR correlates with Hα equivalent width, taken as an indicator of the star formation rate (see, e.g., Gavazzi, Boselli and Scodeggio, 1990). The same authors found that galaxies in clusters have their FIR/optical luminosity functions not distinguishable from those of isolated galaxies. The

most obvious interpretation of this result is that the global star formation process in spiral galaxies is not significantly altered by the cluster environment. On the other hand, the FIR/optical luminosity ratio is enhanced in spiral galaxies belonging to a wide sample of isolated pairs (Sulentic, 1990).

Also the radio continuum emission is found to correlate with Hα equivalent width and, in this case, there is some evidence that spiral galaxies in clusters have their radio emission (per unit visible light) enhanced with respect to isolated galaxies (e.g. Gavazzi and Jaffe, 1986; Gavazzi, Boselli and Scodeggio, 1990). The same holds for interacting and paired galaxies compared to relatively isolated galaxies (Sulentic, 1976; Storne, 1978; Hummel, 1981).

1.7 Rotation Curves

In the last years, recent observations allow us to discuss the possibility that the rotation curves for spiral galaxies also differ in different environments. Several papers have addressed the previous issue with contradictory results (Rubin 1983, Chincarini and de Souza 1985, Guhathakurta et al. 1988, Burstein et al. 1986). In a recent work, Whitmore et al. (1988) found a good correlation between the outer velocity gradient [1] of the rotation curve and the galaxy distance from the center of the cluster, in the sense that rotation curves of galaxies in the central region of clusters are generally falling, while the outer galaxies in a cluster and field galaxies tend to have flat or rising rotation curves (see Fig. 1.8). This correlation indicates that the inner cluster environment can strip away some fraction of the mass in the outer halo of a spiral galaxy or, alternatively, may not allow to form the outer halo. An opposite result was obtained by Balkowski (1990) who presented Hα velocity fields of galaxies in different clusters and obtained velocity curves in order to test the influence of the environment: she found that the gradient of rotation curves does not depend on the environment. However, the velocity gradient is also related to the galaxy luminosity (Persic and Salucci, 1988). A recent paper, in which correlations between gradients, environmental-density, galaxy luminosity and arm-class are discussed, suggests also the presence of an intrinsic gradient – arm-class correlation (flat or decreasing rotation curves are

[1] the percentage increase of the rotation curve between 0.4 R_{25} and 0.8 R_{25}, normalized to the maximum rotational velocity

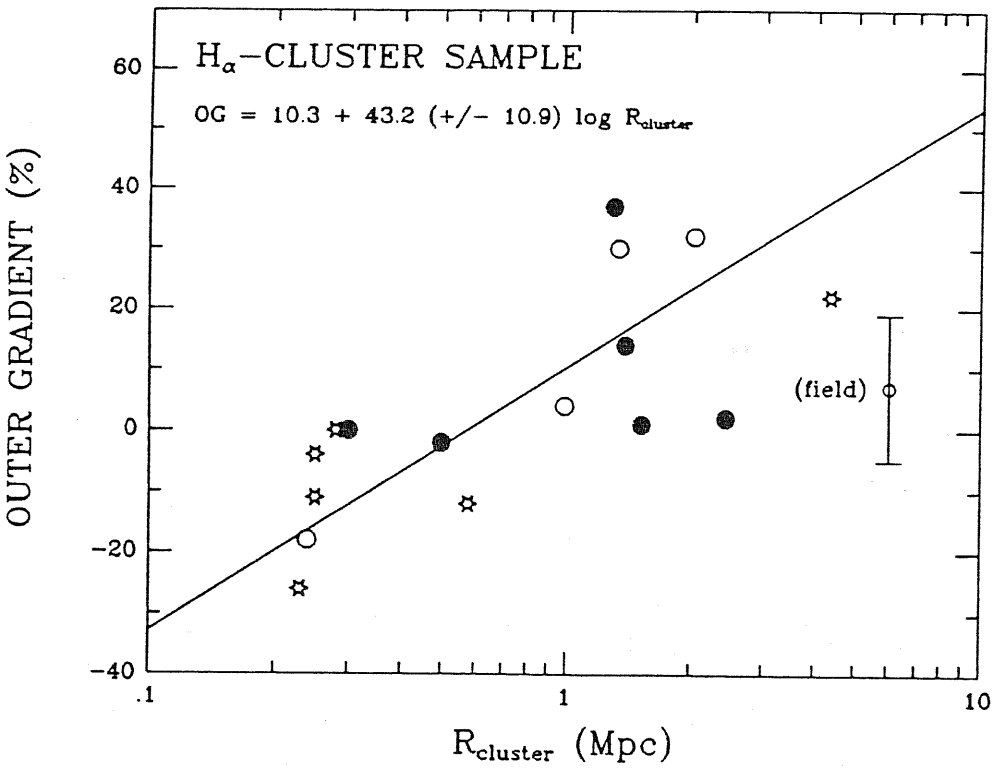

Figure 1.8: Outer gradient of the rotation curve (percentage increase from $0.4R_{25}$ to $0.8R_{25}$, normalized to V_{max}) vs. the projected distance from the center cluster for the Hα cluster galaxies. Open circles are Sa galaxies; filled circles are Sb galaxies; open stars are Sc galaxies. The average value for the Hα field sample included with error bars showing the 1 σ scatter of the distribution. Galaxies near the centers of the clusters have falling rotation curves. (from Whitmore, 1988)

preferentially found in spirals with grand design arms, rising rotation curves are found in flocculent galaxies), see Biviano et al. 1990a.

1.8 The Tully-Fisher relation

The universality of the Tully-Fisher relation, that is the linear correlation between the logarithm of the 21-cm line-width and the photographic absolute magnitude, is an open issue. Bothun et al. (1984) claimed there are similar IR Tully-Fisher relations in the field and cluster environments; so did Richter and Huchtmeier (1984) and Giuricin, Mardirossian and Mezzetti (1986) in the Blue band. On the contrary, Roberts (1978) and Rubin et al. (1985) found different relations in galaxy samples of different morphological type. Djorgovski, De Carvalho and Han (1989) have questioned the universality of some distance-indicators, among which the Tully-Fisher relation. In a very recent work, Biviano et al. (1990b) founnd that there is no strong environment-dependence. A similar result is obtained by Burstein (1990) who confirmed that the two distance indicators, the Tully-Fisher and D_n-σ relations, do not appear to vary with density of environment. The author claimed that all of these facts lead one to a picture of galaxy formation in which the formation process is heavily influenced by the environment (see morphology and HI content) of the galaxy, but that the internal working of a galaxy, once formed, is dictated more by the gravitational field of the galaxy than by external influences.

1.9 Size and Luminosity

The luminosity of galaxies may also depend on the environment. In general, it has been claimed that galaxies of higher luminosity may slightly prefer denser environments (e.g., Binggeli, 1987). As a matter of fact, bright galaxies in cluster appear to be brighter than non-cluster bright galaxies (e.g., Chincarini, 1990) and the absolute magnitudes of the brightest (relatively) isolated galaxies turn out to be, on average, one magnitude fainter than those of the clustered brightest galaxies (Einasto and Einasto, 1987), although the overall galaxy luminosity function is substantially independent of the environment (e.g., Chincarini, 1990 and references therein). Other observational evidence of a general slight total luminosity segregation affecting the whole galactic

population as well as individual morphological types, has recently been reported by Xia et al. (1986,1988), Giovanelli and Haynes (1988), Hamilton (1988). Iovino et al. (1990) find that not only at a given luminosity, there is an explicit dependence of the clustering properties on the morphological type, but even that clustering properties are related to the luminosity for each morphological type. So, according to Iovino et al. (1990) there are brighter objects in regions of higher density, and this is not singly due to the morphology-density relation, but it is rather an independent fact in the distribution of galaxies. As regards the ellipticals galaxies, dwarf ellipticals also show a dependence on environment: Ichikawa et al. (1988) found that dE's in the central region of the Virgo cluster have larger diameters and brighter magnitudes than those in the outer region.

Concerning the disk galaxies, however, there is no agreement about the possible relation between the luminosity of galaxy bulge and the environment. Dressler (1980) found that bulges of both S0 and S galaxies tend to be more luminous in higher-density regions. On the other hand, Solanes, Salvador-Sole', and Sanroma' (1989), using a similar sample but bulge-magnitude complete, found no dependence of bulges, for fixed type, on environment (see Fig. 1.9).

In a discussion not restricted to disk galaxies, one of the best indications that the cluster environment can affect the structure of galaxies is the existence of cD galaxies near the centers of many clusters. They are clearly distinguishable from ordinary ellipticals both because they are so much brighter (e.g. Oemler 1976) and because their halos are enormous compared to the halos of ellipticals (e.g. Morgan and Lesh 1965; Oemler 1973). Several explanations have been proposed for the existence of cD galaxies (see e.g. Kormendy 1982): for instance Richstone (1976) proposed a picture in which tidal stripping by galaxy encounters and by cluster potential (e.g. White 1982) produces a galactic sea of stars which is identified as the cD halo; so we might expect other galaxies in the cluster to have lost material in order to feed the cDs. However, it is clear that cDs have grown in size as a result of the cluster environment (cDs exist only in clusters), so it seem important to look for dependence of galaxy size on environment. Kormendy (1977b, 1979, 1982) found a number of elliptical galaxies which show a luminosity excess in their outer regions in comparison to a de Vaucouleurs law fitted to the inner regions. Schombert (1986) found that faint ellipticals in his sample have outer cutoffs in their profiles. Several other photometric studies,

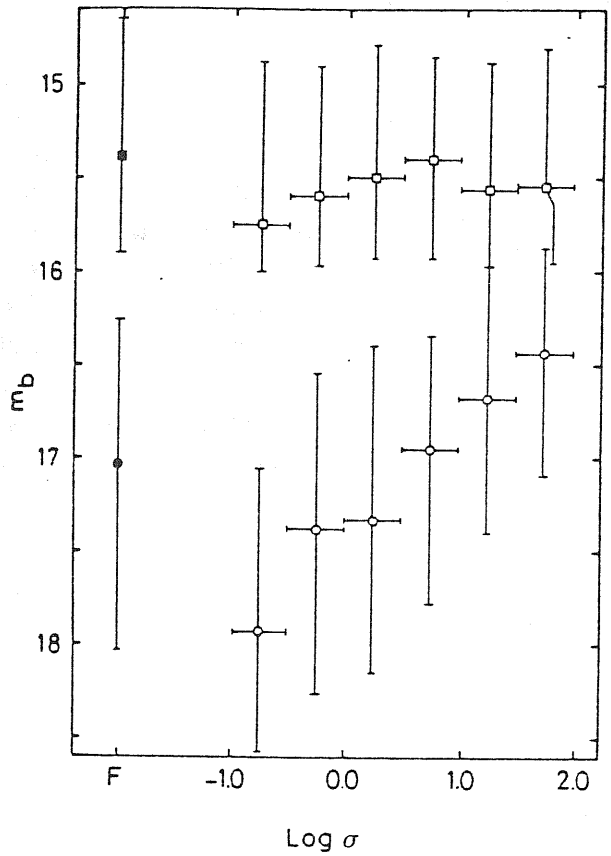

Figure 1.9: Median and quartile bulge magnitudes versus local density for disk galaxy population. Circles and squares correspond to the total- and bulge-magnitude-complete samples, respectively. Field-sample values are given by closed symbols. To avoid any spurious effect due to density uncertainty, only those points arising from more than nine galaxies are included. (from Solanes, Salvador-Sole' and Sanroma', 1989)

including Bothun and Schombert (1988), and Lauer (1988), have also found evidence of truncations in galaxy profiles. On the other hand, these photometric studies are very impegnative and, consequently, limited to a small number of galaxies. Measures of luminosity and size exist for larger samples of galaxies, so it is usefull to compare the size of the galaxies for a fixed luminosity. E.g., Strom and Strom (1978a,b,c) found that the radii of the elliptical galaxies (measured at the 26th mag arcsec^{-2} isophote in the R band) in the dense spiral-poor clusters are about 30% smaller at a given M_V than the corresponding radii in the spiral-rich clusters; a similar result holds for spiral galaxies (Peterson, Strom and Strom, 1979). Moreover, a well-defined relation between galaxy luminosity and absolute diameter exists (see chapter 2). This relation is so tight to seem a very suitable tool to study the influence of the environment on galaxies. So, it is useful to compare the trends of luminosity-diameter relation for galaxies setted in different environments. The results of the relating literature are further described in chapter 4.

Chapter 2

The Luminosity-Diameter relation

Summary

The study of the relation between galaxy intrinsic luminosity and absolute diameter (the L-D relation) require a precise definition of the luminosity and diameter, and of the corrections to apply them. A well-defined L-D relation for disk galaxies is expected to exist since surface brightness of disk galaxies follows an exponential law, and the central surface brightness is roughly constant for all disk galaxies. Many authors studied the trend of the L-D relation (which is linear in its logarithmic form: magnitude-log diameter) for galaxies of different morphological types. Moreover, the L-D relation does not seem to be a good distance indicator. Several authors discussed the possibility that the trend and the dispersion of the L-D relation are biased by selection effects. So long, the issue of the selection effects remains an open question. However, probably the L-D relation is not or is less affected by selection effects in the range of high luminosities.

2.1 Magnitudes and Diameters

A quantitative approach to galaxy properties starts from the determination of the projected surface brightness distribution $\sigma(r, \theta)$ as a function of the polar coordinates r and θ, taking the origin in the center of the galaxian

image and referring the position angle to a given direction. This function give us a lot of informations on the physical state of the galaxy: its total light output, shape and size.

The surface brightness distribution is generally expressed as $\sigma(r,\theta) = \sigma_0 F(r,\theta)$, where $\sigma_0 = \sigma(0,\theta)$ is the central value, F is an dimensionless function of the distance r from the center of the galaxy image, and of the azimuthal angle θ. When one computes the total luminosity of a galaxy , a simpler version of σ is adopted: it is the so called *circularized equivalent radial brightness profile* $\sigma(r) = \sigma_0 f(r)$, where

$$f(r) =< F(r,\theta) >_\theta = 1/2\pi \int_0^\infty F(r,\theta) d\theta \tag{2.1}$$

and

$$\sigma(r) =< \sigma(r,\theta) >_\theta \tag{2.2}$$

so the total luminosity is

$$L_T = \int_0^\infty \int_0^{2\pi} \sigma_0 F(r,\theta) r dr d\theta = 2\pi\sigma_0 \int_0^\infty f(r) r dr \tag{2.3}$$

Surface photometry (see e.g. Sandage 1961, de Vaucoleurs and de Vaucouleurs 1972) showed that the brightness distribution [1] in normal galaxies may be resolved into two major components:

i) a spheroidal component characteristic of elliptical galaxies (Hubble 1930, de Vaucouleurs 1948a, etc)

$$log\sigma(r) = log\sigma_e - 3.33[(r/r_e)^{1/4} - 1] \tag{2.4}$$

where σ_e and r_e are a brightness and a size scale: r_e is the radius that contains half of the total light, and σ_e is the brightness at r_e.

ii) a flat or exponential component characteristic of the disk of late-type spirals (de Vaucouleurs 1958a,etc.) and of Magellanic irregulars (Ables 1971, de Vaucouleurs and Freeman 1972)

[1] Unless otherwise stated, all optical intensities pertain to the photographic B-band and all luminosities are based on the B_T^0 magnitude system, which includes corrections for both inclination and galactic extinction.

$$\sigma(r) = \sigma_0 e^{-r/r_0} \tag{2.5}$$

where σ_0 is the extrapolated central surface brightness and r_0 is the scale lenght. I will refer to the equations (2.4) and (2.5) as de Vaucouleurs' law and exponential law.

Ellipticals obey very closely equation (2.4), Magellanic irregulars equation (2.5); intermediate stages of the Hubble sequence are well represented by linear combination of (2.4) and (2.5). The two most fundamental components of spiral galaxies are a spheroidal or ellipsoidal distribution of stars (bulge), following the (2.4), and a flat distribution of stars, following the (2.5). However significant differences has recently been found between bulges and ellipticals, both in the photometry structure (Kormendy 1980, Boronson and Kormendy 1982, Capaccioli 1987), and in their dynamical properties. The relative importance of the bulge with respect to the disk, that is the B/D ratio, appears to decrease smoothly along the Hubble sequence (de Vaucouleurs, de Vaucouleurs and Corwin 1976, hereafter RC2): in order to have a good estimate of this ratio, de Vaucouleurs (1958a) first recognized the need to decompose the observed light profiles into spheroid and disk components (see also Kormendy 1977a, Burstein 1979, Simien and de Vaucouleurs 1986). An example of decomposition of the observed light profile is presented in Fig. 2.1 .

Photometric data of the quality needed to determine such a detailed brightness distribution are available for few galaxies. For most of the galaxies, only of brightness and angular scale are available.

Three types of integrated magnitude are available in the literature: metric magnitudes, isophotal magnitudes, and total magnitudes. Photoeletric measurements lead to estimates of the light contained within a certain aperture; these are called *metric magnitudes*. Because the metric absolute magnitudes of the brightest galaxies in rich clusters of galaxies show a remarkably small dispersion, photoeletric metric magnitudes play an important role in attempts to detect the deceleration of the expansion of the Universe. Photographic photometry, by contrast, leads naturally to *isophotal magnitudes*, i.e., estimates of the light contained within a certain isophotal contour. For example, Holmberg (1958) has obtained magnitudes interior to the 26.5 pg mag arcsec^{-2} contour fo several hundred objects. *Total magnitudes* are those estimated from metric or isophotal magnitudes by some sort of extrapolation procedure. In principle, these magnitudes represent the total amount of light emitted by a galaxy, but it is essential to remember that the bright-

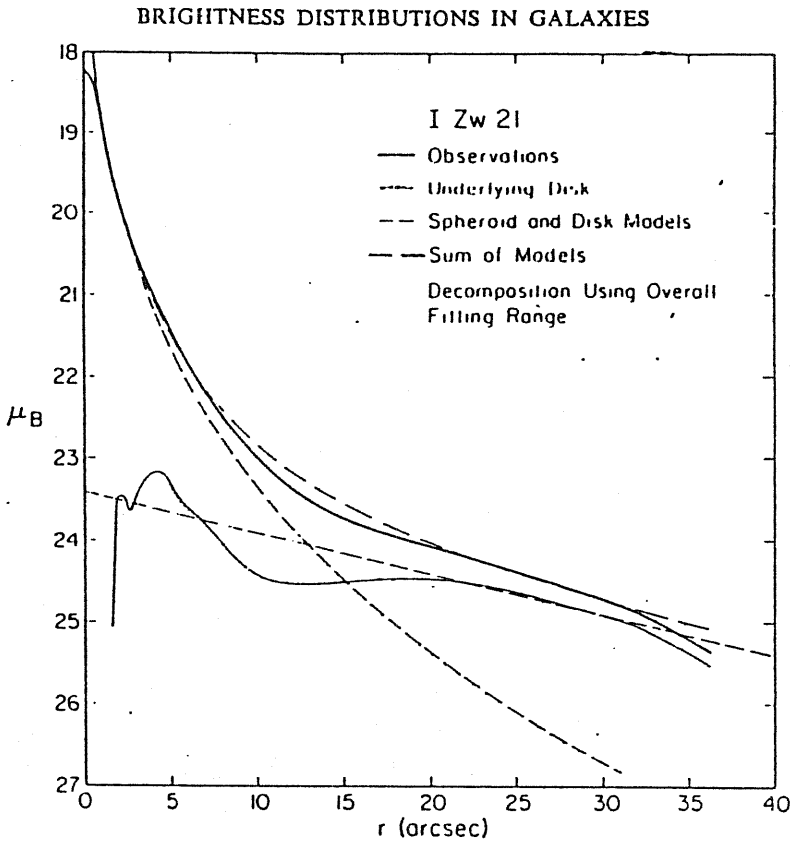

Figure 2.1: Decomposition of I Zw 21, using the iterative fitting algorithm: surface brightness in B-mag arcsec^{-2} versus radius in arcsec. (from Kormendy, 1977a)

ness profiles of many galaxies drop so gradually that extrapolation can be both dangerous and strongly model dependent. Nonetheless, the most widely quoted magnitudes, those of the *Second Reference Catalogue of Bright Galaxies* (RC2), are total magnitudes in this sense. The introduction to this catalog describes the extrapolation procedures adopted and gives extensive bibliography for the sources of galaxian magnitudes.

The presence of dust in galaxies, generally spirals or irregulars, can both scatter light from its original path and absorb it, in such a way that blue light is more strongly absorbed and scattered than red light. So, the observed total magnitudes (and integrated colours) of galaxies suffer from two kinds of extinction effects: the galactic absorption and that one produced by internal obscurations in the galaxies. Both effects can be determined, at least in a statistical sense, by an analysis of the observational data. The galactic extinction has been rediscussed by many authors, but there is not a general agreement on the correction to be applied.

The absorption due to our Galaxy can be described by a cosecant law of this kind

$$galactic\ absorption = \alpha\ cosec\ |\ b\ |\qquad mag, \tag{2.6}$$

where b is the galactic latitude and α is a coefficient of proportionality (the amount of polar absorption). This formula corresponds to the assumption of a plane parallel dust distribution, with dust density depending only on distance from the galactic plane. This is certainly a rough oversemplification of the dust distribution in the Galaxy. A more sophisticated formula is used in the RC2 in which α varies with galactic latitude and longitude according to a formula depending on 21 parameters. This formula has been determined from galaxy counts, bright galaxy colours, and optical-to-radio-emission ratios. But RC2 formula predicts about 0.2 mag of extinction in the polar caps, whereas there is a substantial evidence for essentially zero extinction toward the north galactic pole and little toward the south galactic pole. Sandage proposed in 1973 an ad hoc modification to take account of the extinction-free polar caps. Another widely used formula is that one proposed by Fisher and Tully (1981).

As regards the internal absorption, dust tends to make disk galaxies fainter when seen edge-on, because in this case more light is absorbed and scattered away from the line of sight by the dust. Therefore a galaxy looks

brighter when viewed face-on. The face-on magnitude is the standard measure of intrinsic luminosity . Since the absorption in the Galactic system can be statistically described by a cosecant law, a similar law may be expected in external spiral systems: the latitude is replaced by the inclination i [2] between the line of sight and the plane of the galaxy disk.

$$internal\ absorption = const. \cdot cosec\ i \qquad mag \tag{2.8}$$

Different authors proposed different corrections for inclination.

Another effect which must be taken into account is the so called *K correction*, which describes the dimming of light from the galaxy caused by its recession from us. Already in 1936, Hubble took account of a third correction term, the term K, which allowed for the fact that the shift of the spectrum of a distant galaxy to the red brought a different band at shorter wavelengths (measured in the rest frame of the receeding object) into the spectral acceptance band of the observer's photographic plate. The expression for the K-correction has two terms (Humason, Mayall, and Sandage 1956; Oke and Sandage 1968): i)The acceptance band is narrower in the rest frame of the distant object by a factor $(1+z)$, independent of wavelength or of the shape of the energy curve; the apparent magnitude of the object is thereby increased by an amount $2.5log(1+z)$. ii)At each wavelength λ in the acceptance band the intensity received is not the $F(\lambda)$ read from the standard energy curve measured in the observer's velocity frame, but $F[\lambda/(1+z)]$. The apparent magnitude will be increased if the energy curve falls toward shorter wavelengths, or decreased if it rises. The effect of redshift on the integrated B magnitudes has been precisely derived by Pence (1976) for all normal galaxy types. For $z \ll 1$ RC2 proposed a simplified relation.

In conclusion, in order to obtain standard corrected magnitudes it is necessary to apply three correction: 1) the interstellar absorption within our

[2]The inclination i of the polar axis of a galaxy can be related to the ratio d/D of the apparent axes $D = 2a$, $d = 2b$ of the outer isophotes of the nebular image, assuming that the isophotal surfaces of the galaxy can be approximated by ellipsoids of revolution of true axis $q_0 = c/a$ which project as ellipses of ratio $q = b/a$. The relation is

$$cos^2 i = (q^2 - q_0^2)/(1 - q_0^2) \tag{2.7}$$

The relation between i and q can be obtained selecting for q_0 the observed value for the most flattened galaxies of each morphological type (table 1, by Heidmann, Heidmann and de Vaucouleurs 1972.)

own Galaxy, 2) the internal galaxian absorption, and 3) the K correction.

Due to the low density gradients in the outermost parts of galaxies, the definition of the major and minor diameters encounters certain difficulties. The problem may be simplified by assuming either that the diameters should be referred to a given isophote (*isophotal* or *brightness* diameters) or that the boundary defined by the diameters should enclose a certain portion of the total luminosity. In the first case diameters are derived from direct measurements of photographic images; the second definition demands a determination of the surface luminosity distribution in each object. The Holmberg diameter, referring to a 26.5 pg mag arcsec^{-2} isophote, and the most used R_{25}, referring to 25 B-mag arcsec^{-2} isophote, are examples of the first definition. The second definition has been utilized, for example, by de Vaucouleurs(1948b): the *effective* major and minor diameters define an ellipse enclosing half the total luminosity of the galaxy.

Apparent diameters that are measured directly on photographs may be divided in two groups: those that are determined by visual inspection of the plate (usually called *estimated* diameters), and those that are derived from photometric tracings (*photometric* diameters). Because the human eye is an imperfect instrument in recording diameters (the eye overestimates the length of an elongated image and underrates its width), a systematic error exists in estimated diameters, so they are less precise than the photometric diameters that, nevertheless, include certain systematic effects. In fact all brightness diameters, whether estimated or photometric, are dependent on the galactic absorption and on the inclination of the galaxy to the line of sight. On the other hand, the effective diameter is an intrinsic quantity that uniquely characterizes the linear scale of a galaxy obeying a specific law of luminosity distribution; in particular it is indipendent of density or emissivity and almost indepent of inclination, except for absorption effects.

The variation of isophotal diameter with inclination or axis ratio is derived for a transparent spheroidal galaxy obeying a r^{-n} law of emissivity distribution (for details, see Heidmann, Heidmann and de Voucouleurs 1972, hereafter H^{2}V). Fixed R (the axis ratio), D and $D(0)$ (the isophotal diameter and the isophotal "face-on" diameter, respectively) the following relation is obtained (H^{2}V)

$$logD(0) = logD - \frac{1}{n-1}logR = logD - ClogR \qquad (2.9)$$

where a precise value of C can be calculated theoretically and empirically verified (e.g. table 7, H^2V). The presence of internal absorption has a negligible effect on the above relation. However, there are different opinions: Tully (1972) questioned on the effects of inclination on observed diameters of galaxies and concluded that diameter corrections do not have to be made. For the general accepted formulas see RC2, which is in agreement with H^2V.

The apparent diameters of galaxies are reduced by galactic extinction. The effect may be calculated for galaxy models obeying the r^{-n} law; the relative variation of the semi major axis a_0, that is $\delta a/a$, may be computed (e.g. table 7 H^2V). If the galactic absorption A (in mag) is small, so that $A\delta a/a \ll 1$, the corrected semi major axis a_0 is given by

$$log\ a_0 \simeq log\ a + 0.43A\frac{\delta a}{a} \tag{2.10}$$

An analogous formula holds for corrected diameters (see RC2 for the value of G).

$$logD(0) = logD + AG \tag{2.11}$$

At the end, using distance information, it is possible to obtain the absolute magnitude and the absolute diameter. The transformations from apparent magnitude m (mag) and diameter D (0.1 arcmin) to absolute quantities M and D (Kpc) are:

$$D(Kpc) = 2\ 10^3\ d \cdot tan[1/2\ \frac{\pi}{180}\frac{D(0'.1)}{60}] \tag{2.12}$$

$$M = m - 5log\ d - 5 \tag{2.13}$$

where d is the distance in Mpc.

2.2 The Luminosity-Diameter relation

All the following discussion about the luminosity-size relation may be summarized by the conclusion of Freeman (1970), who states that a strong correlation must exist between the absolute magnitude M_B and the length scale a^{-1} because (i) the disk usually provides most of the blue light in spiral and S0 systems, (ii) the total luminosity of the exponential disk is $L_T = 2\pi\sigma_0/a^2$,

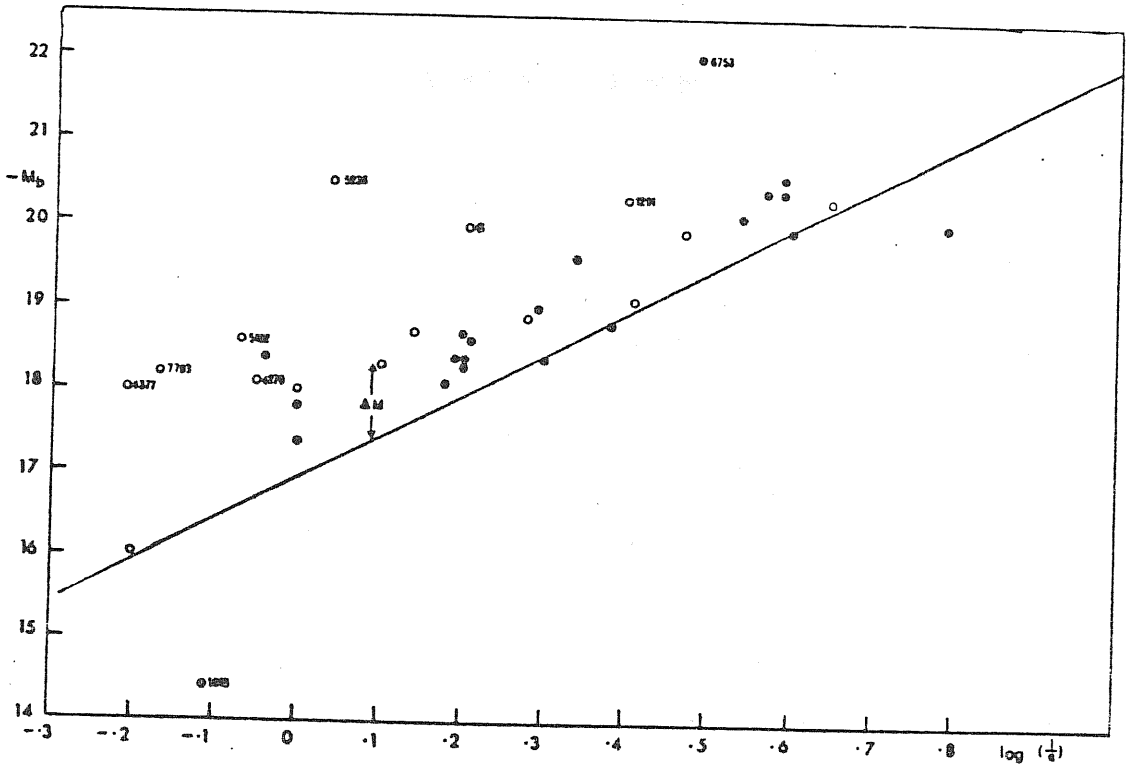

Figure 2.2: Absolute magnitude M_B against the logarithm of the length scale a^{-1} (kpc). Straight line represents $[M_B,\ log(a^{-1})]$-relation for exponential disks with central B-surface brightness $B(0)_c = 21.65$ mag per square second of arc. (from Freeman, 1970)

(iii) the central surface brightness σ_0 is nearly constant for most spiral system. Figure 2.2 shows M_B against lenght scale a^{-1}; the expected correlation is present. The straight line is the theoretical relation for an exponential disk with S_0=21.65 mag arcsec^{-2} (S_0 is σ_0 expressed in mag arcsec^{-2}). Since the spheroidal component contribuites to the absolute magnitude M_B, most of the points lie above this line. Now I will discuss the luminosity-diameter relation (hereafter L-D relation) in detail.

If one assumes de Vaucouleurs' and exponential laws (2.4 and 2.5) for the galaxy luminosity profiles, it is possible to write a single expression for both ellipticals and pure disks (see de Vaucouleurs 1962 and references therein). The mean radial distribution of the surface brightness $\sigma(r)$ can be modeled by

$$\sigma(r) = \sigma_0 exp[-(r/a)^{1/n}], \tag{2.14}$$

where $n = 4$ for ellipticals (and central bulges of spiral) and $n = 1$ for disk of spirals;

In (2.14) r is the radius, σ_0 is the central surface brightness, and a is a scale length which is related to the effective radius R_{eff}

$a = 2.891 \cdot 10^{-4} R_{eff}$ for ellipticals

$a = 0.5958\ R_{eff}$ for disks of spirals

The total luminosity L_T is then

$$L_T = \int_0^\infty 2\pi r \sigma(r) dr = (2n)! \cdot \pi \sigma_0 a^2 \tag{2.15}$$

so that

$$L_T \propto \sigma_0 R_{eff}^2 \tag{2.16}$$

If it is possible the following decomposition $\sigma_0 = \sigma R_{eff}^p$, where σ does not depend on the size (but may depend, e.g., on the morphological type), then the size-luminosity relation will be

$$L_T \propto \sigma R_{eff}^{p+2}, \tag{2.17}$$

or, in the logarithmic form,

$$logL_T = const. + log\sigma + (p+2)logR_{eff} \tag{2.18}$$

So, in the plane ($logR_{eff}$, $logL_T$), the slope of the straight line is given by $p+2$, but the dispersion is given by the range of possible value for σ.

Now, I will limit the discussion to disk galaxies. In 1970 Freeman collected the best photometry then available for 36 disk galaxies and found that for about three-quartes of the sample (28 out of 36), the corrected central Blue surface brightness S_0 varies little from $< S_0 >= 21.65$ (mag arcsec^{-2}) and is indipendent of morphological type (from $T = -3$ to $T = 10$) (Fig. 2.3). After this previous work, there were several evidences (e.g. Boronson, 1981; van der Kruit, 1987) that the central surface brightness for disk galaxies is nearly constant, even if it is possible that this founding is produced by observational selection effects (see section 2.5). However, if σ_0 is assumed to be a constant, equation 2.16 becomes

$$L_T \propto R_{eff}^2. \tag{2.19}$$

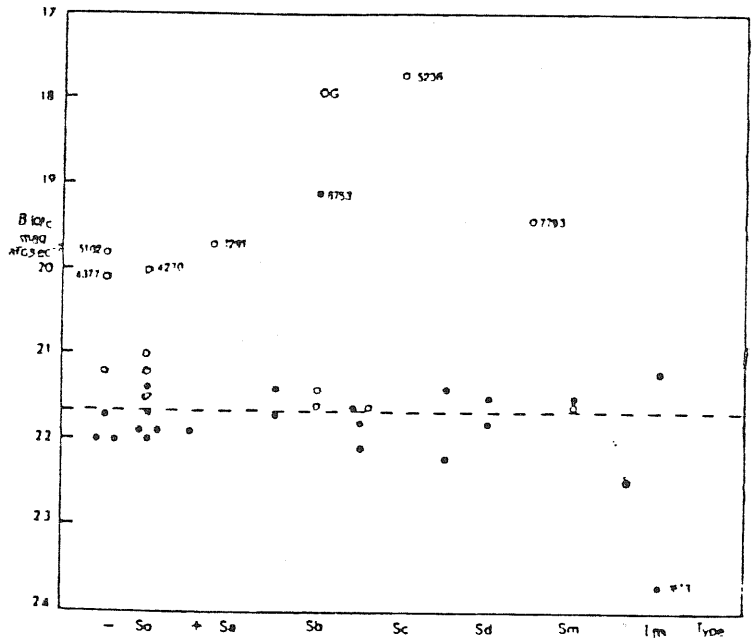

Figure 2.3: Central B-surface brightness $B(0)_c$ for the exponential disks of thirty-six galaxies against their morphological type. Broken line at $B(0)_c = 21.65$ is the mean for twenty-eight galaxies. NGC numbers are shown for the other eight. G denotes an estimate for the Galaxy. *Filled circles*, Type I luminosity profile; *open circles*, Type II luminosity profile, see the original paper for details. (from Freeman, 1970)

Moreover, from equation 2.14 (with n=1 for disk galaxies) , the radius R_{25} , referring to a 25 mag arcsec^{-2} isophote is equal to $-a \cdot ln(\sigma_{25}/\sigma_0)$, $a \propto R_{eff}$ and so the previous equation 2.19 may be written as

$$L_T \propto R_{25}^2 \tag{2.20}$$

with a different constant of proportionality. In the same way it is possible to consider any other photometric radius (e.g. R_{26} [3]).

Obviusly also the corrispondent equations with the diameters are valid

$$L_T \propto D_{eff}^2 \tag{2.21}$$

$$L_T \propto D_{25}^2 \tag{2.22}$$

For a pure exponential disk galaxy which obey *perfectly* to the exponential law, the expected diameter-luminosity relation in its logarithmic formula is

$$logL_T = const. + 2logD_{eff} \tag{2.23}$$

that is, a relation with slope=2.

In general, for disk galaxies, also the bulge must be considered, particularly for early types. The effect of the bulge is not easily predictable. First, the measured L_T is the sum of the pure disk luminosity L_D and of the bulge luminosity L_B, and in the literature several estimates of L_D/L_B exist. Second, the size is determined by both disk and bulge: only for late type spirals the diameter is definited by the disk "only". Third, as reported by Kormendy (1982), the characteristic surface magnitude of galaxy bulges is not a constant (see Fig. 2.4). More in general, as hinted in the previous section, it is no clear if de Vaucouleurs' law for ellipticals is valid for also bulges.

Not only Freeman (1970), but several authors have discussed various correlations between linear diameter and luminosity. Pioneering work reported the correlation between absolute or apparent magnitude and linear or angular diameter (see Hubble, 1926; Heidmann, 1967).

Analysing of Holmberg's (1964) photometric data on spiral galaxies and Liller's (1960, 1966) photometric data on elliptical galaxies of the Virgo cluster, Heidmann (1967) showed that there is a very close relation between the

[3]the radius at 26 mag arcsec−2 isophote

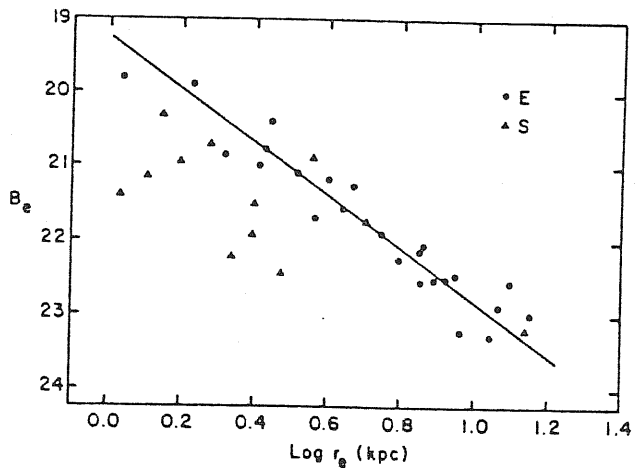

Figure 2.4: Comparison of the $B_e - log\ r_e$ relations for $r^{1/4}$-law fits to the mean profiles of bulges and ellipticals (from Kormendy 1980), B_e is B-surface brightness at the effective radius r_e. Bulges which contribute most of the light of their galaxies are consistent with the relation (straight line) for ellipticals. Bulges such as M31 and M81 have larger r_e and fainter B_e than ellipticals of the same luminosity-lines of constant luminosity have slope 5 and are therefore steeper than the line shown for ellipticals.

intrinsic luminosity L of a galaxy and its absolute diameter D up to a faint (fixed) isophote of the form

$$L \propto D^q \tag{2.24}$$

The best value of q is 2.8 for spirals and 1.9 for ellipticals. The equation 2.24 is valid over a 5 magnitude range with a rms deviation for D equal to only 11 per cent. According to Heidmann (1967) the value of q does not depend on the limiting isophote for the diameters D.

Sersic (1968) discovered that the diagram of effective radius versus blue-magnitude (M_B) for elliptical and S0 galaxies shows a bright-galaxy sequence ($M_B \lesssim$ -20.5), a parallel faint-galaxy sequence ($M_B \gtrsim$-18), and a stubby transition region at -20.5$\lesssim M_B \lesssim$-18 (see also Brooks and Rood, 1971; section 2.3).

In a later paper, Heidmann (1969) reported the relation for galaxies in the Coma cluster (see Fig. 2.5). Relation 2.24 is confirmed, but here the dispersion is much larger than for the Virgo cluster galaxies: q=1.8, close to the value for Virgo ellipticals. Heidmann suggests that the L-D relation is a function of only two parameters: absolute luminosity and morphological type.

Holmberg (1969, see also 1975) reported the correlation (see Fig. 2.6) based on the data listed in Holmberg catalog (1964), and he thus refers to the Holmberg magnitude and diameter system (see previous subsection). The diagram included 189 galaxies of the types Sa-Sb-Sc, 36 of the types E-S0, 16 of the type Ir I, and 5 of the type Ir II, in all 246 objects. Holmberg found a regression line represented by the equation $M = -6.00 \cdot logD + 7.14$ where M is the absolute photographic magnitude and D the absolute isophote diameter at 26.6 B-mag arcsec^{-2} isophote according to the Holmberg magnitude and diameter system. Different type groups do not show any significant systematic deviation from the regression line; the only exception is the E- S0 group.

H^{2}V also found a very tight linear correlation between the log of the face-on photometric diameter and the face-on magnitude in the Virgo Cluster for spiral galaxies: $< q >= 2.6 \pm 0.1$ [4] and no significant variation with morphological type (see also section 2.4). The authors used corrected magnitudes and diameters, according to the studies presented in the paper; on

[4] q is the same as in equation 2.24.

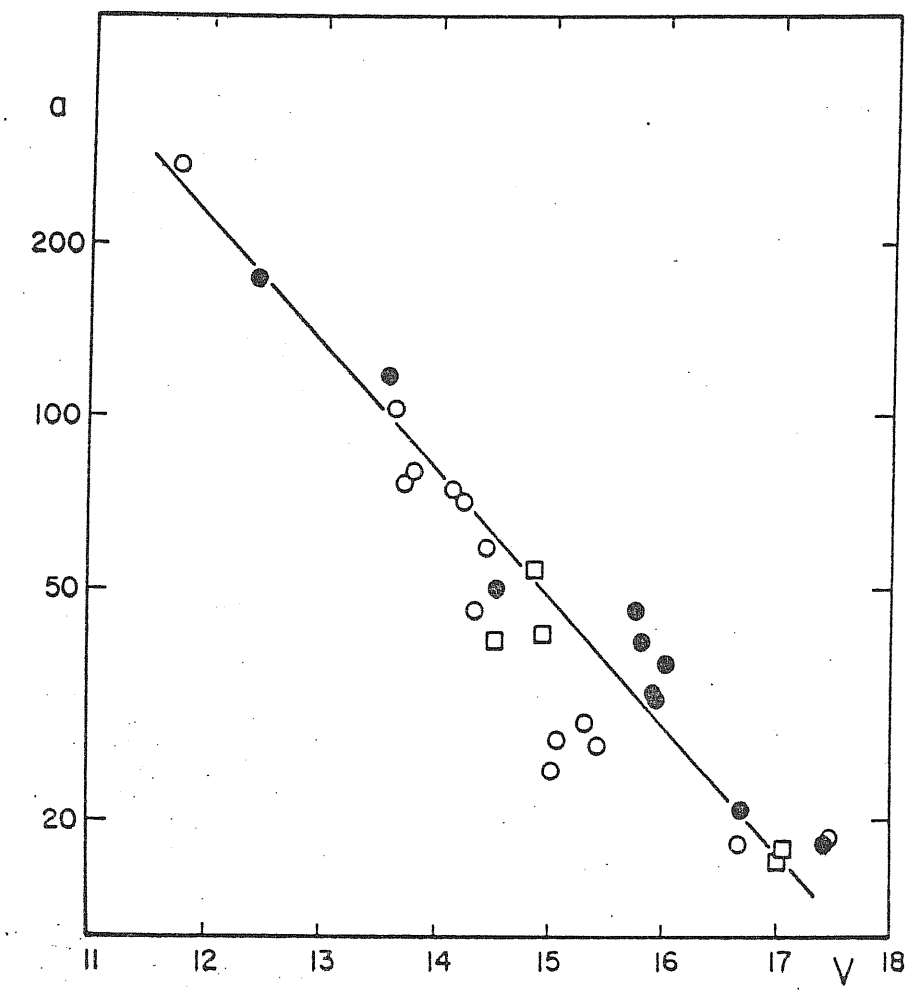

Figure 2.5: Diameter a in seconds of arc, of isophote 26 V-mag arcsec^{-2} as a function of apparent total magnitude V, for Coma cluster galaxies. *open circles*: elliptical galaxies; *filled circles*: S0 galaxies; *open squares*: SB0 galaxies. (from Heidmann, 1969)

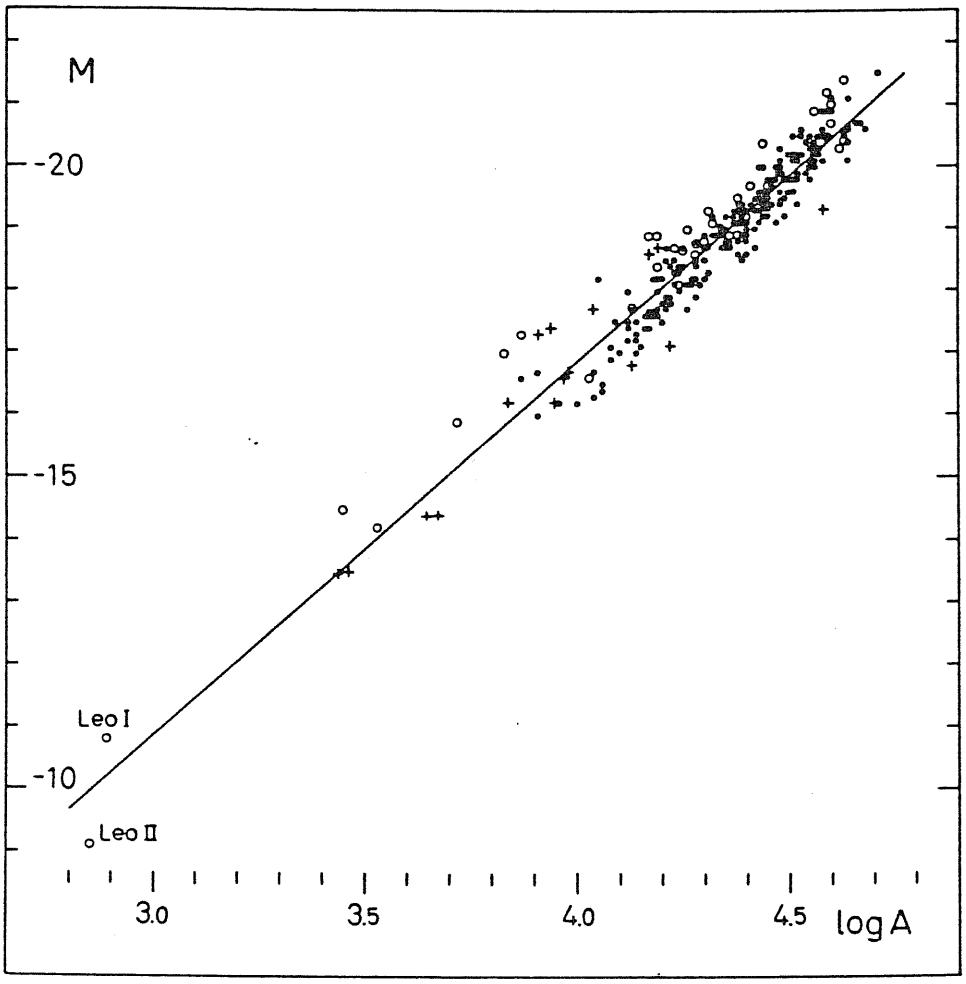

Figure 2.6: Correlation between absolute photographic magnitude and log absolute major diameter (pc), as derived for galaxies of type E-S0 (*open circles*, types Sa-Sb-Sc (*filled circles*), and type Ir (*crosses*). (from Holmberg, 1975)

the contrary Heidmann and Holmberg in the previous papers corrected only magnitudes (luminosities).

Other works on L-D relation will be presented in the following sections.

2.3 Distance Moduli from L-D Relationship

The apparent luminosity [5] and apparent diameter scale as $L_a \sim D_a^2$ when one places an object at various distances d, because $L_a \propto d^{-2}$ and $D_a \propto d^{-1}$. So, if one assume the validity of relation $L \propto D^q$ between intrinsic luminosity and absolute diameter, measurement of the angular size and the apparent magnitude of a galaxy could therefore be used to determine its distance. The distance modulus μ of a galaxy may be derived from its apparent magnitude m and apparent diameter D_a by

$$\mu = \frac{m}{1-q/2} + 2.5\,\frac{q}{1-q/2}\,logD_a + K \tag{2.25}$$

where K is a constant, obtained by an absolute calibration using distance moduli known from primary and secondary indicators (see Rowan- Robinson, 1985, for the problem of distance determination). The solution degenerates for q=2 and no distance information is contained: the slope and the intercept of the magnitude-log(diameter) relation are not dependent on distance! If the value of q differs significantly from 2, it is possible to derive the distance modulus.

Tully (1968) suggested the possibility to use the L-D relation to determine the distance.

Brooks and Rood (1971) presented radius versus magnitude diagrams for E and S0 galaxies in a central zone of the Coma cluster. As Sersic (1968), they also show bright and faint parallel sequences and a transition region (see Fig. 2.7). They suggest that the radius versus magnitude relation for E and S0 galaxies, with its distinctive transition region, may provide a valuable distance indicator.

Successively, several authors used the method described by the eq.2.25; H^2V applied this method to Virgo cluster. There is a long series of papers on this subject by Paturel (1975a-b, 1977, 1979). Paturel (1979) examined the

[5] that is, if L is the intrinsic luminosity of a galaxy at a distance d the apparent luminosity is $L/(4\pi d^2)$

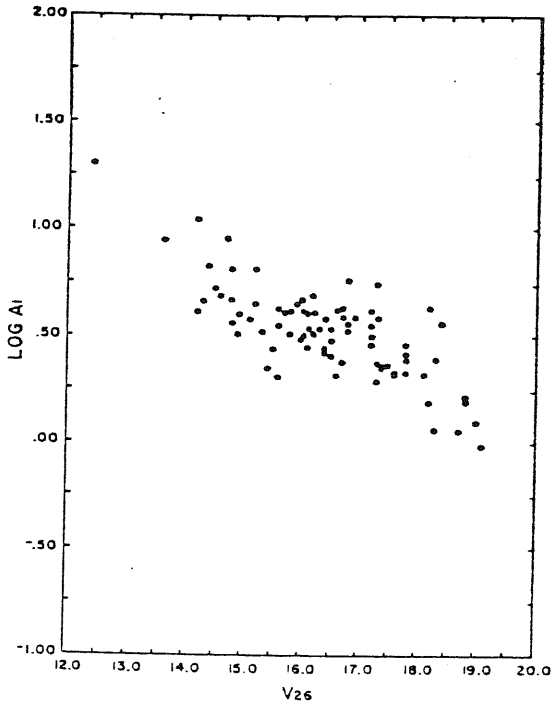

Figure 2.7: Radius-magnitude diagram for 81 S0 galaxies in a central zone of the Coma cluster. Integrated apparent photovisual magnitude (V_{26}) is plotted against log A_1 (the semi-major axis of the isophote within which is contained half of the luminosity represented by V_{26}) in arcsec. (from Brookes and Rood, 1971)

problems of obtaining m and $logD$ properly corrected for perturbing effects and of determining q and K without bias. He applied the L-D relation only to spiral galaxies (T$\geq$0), because his sample of ellipticals has $q \sim 2$. The author applied also the results of L-D relation to determine the Hubble constant.

Romanishin (1986) suggested the use of L-D relation as a tool to study the isotropy of the nearby Hubble flow, as pointed out by Kormendy (1977c).

However, Tammann and Sandage (1983) have pointed out that for galaxies of a fixed Hubble type, the surface brightness is almost independent of absolute magnitude, so this is not a very effective distance indicator. In fact, for a galaxy of angular radius R" and surface brightness S, the apparent magnitude is given by

$$m = S - 2.5\ log(\pi R"^2) + const. \tag{2.26}$$

As long as S= const., the radii (diameters) contain no distance information beyond that contained in the apparent magnitudes. Van den Bergh (1981) suggested that the galaxian infrared surface brightness S_{IR} is related to the 1.6 μm infrared magnitude m_{IR} and to the linear diameter D by the relation

$$S_{IR} = m_{IR}(1.6\mu m) + 5logD \tag{2.27}$$

and that it is a potential distance indicator.

On the other hand, in their recent catalogue Lauberts and Valentijn (1989) found that for all 16 morphological types (from -5 to 10) the relation between intrinsic luminosity, L, and absolute diameters, D, is always of the form $L \propto D^{1.7-2.0}$. The authors pointed out the fact that the intrinsic variation of L with D scales almost with the same exponent as the distance effect, even for all morphological types separately, and this makes it pretty hopeless to try to estimate distances on the basis of photometric parameters alone.

2.4 The L-D relation for different morphological types

As I have already mentioned (see section 2.3), Holmberg (1969, 1975) found that different type groups do not show any significant systematic deviation

from the regression line; the only exception is the E- S0 group, with a deviation of about -0.3 mag towards higher surface brightness region.

H^2V studied the correlaton between the log of the face-on photometric diameter $a(0)$ and the face-on magnitude m_0 for spiral galaxies in the Virgo cluster and looked for a relation more complex than the usual one (eq.2.23), i.e. of the kind

$$log\ a(0) = a + b \cdot T + [c + d \cdot T]m_0 \tag{2.28}$$

where T is the morphological type. The authors calculated the coefficients for a data sample of 25 spiral galaxies (T=1,9) and found that the coefficients of the T terms are not significantly different from zero. However, taking their results at face value, they suggested that the usual fitted line of the L-D relation becomes flatter and flatter when T varies from 1 to 9; this trend is consistent with the very flat fitted line obtained for E($T = -5, -4$) (Heidmann 1969). Giuricin, Mardirossian and Mezzetti (1985b) also found that early-type spirals have a significantly flatter relationship than late-type ones.

As regards the ellipticals, Schombert (1987) showed a plot (absolute V-magnitude M_V against logarithm of effective radius $logr_e$) for BCM (brightest cluster members) ellipticals and normal ellipticals (see Fig. 2.8). In the diagram there is the suggestion of a break in the trend of brighter galaxies, with larger values of r_e around the M_V=-21 point. The break was also suggested by different (L, r)-relations found by Kormendy (1977a) for a sample of bright ellipticals and by Strom and Strom (1979b) which measured a large number of faint ellipticals.

Already mentioned, Lauberts and Valentijn (1989) noted that for all the different 16 morphological types (from -5 to 10) the relation between intrinsic luminosity, L, and absolute diameter, D is always of the same form, i.e. $L \sim D^{1.7-2.0}$. This was established for the 3159 galaxies of the diameter complete sample with known redshift contained in *The Surface Photometry Catalogue of the ESO-Uppsala Galaxies* by Lauberts and Valentijn (1989), hereafter ESO. However, even if the slope of L-D relation for spirals and ellipticals is similar, generally the intercepta is different. Since ellipticals have a higher central surface brightness than spirals, the L-D relation for ellipticals is shifted towards higher surface brightness region in the L-D plane (see e.g Allen and Shu, 1979 and also fig. 2.12).

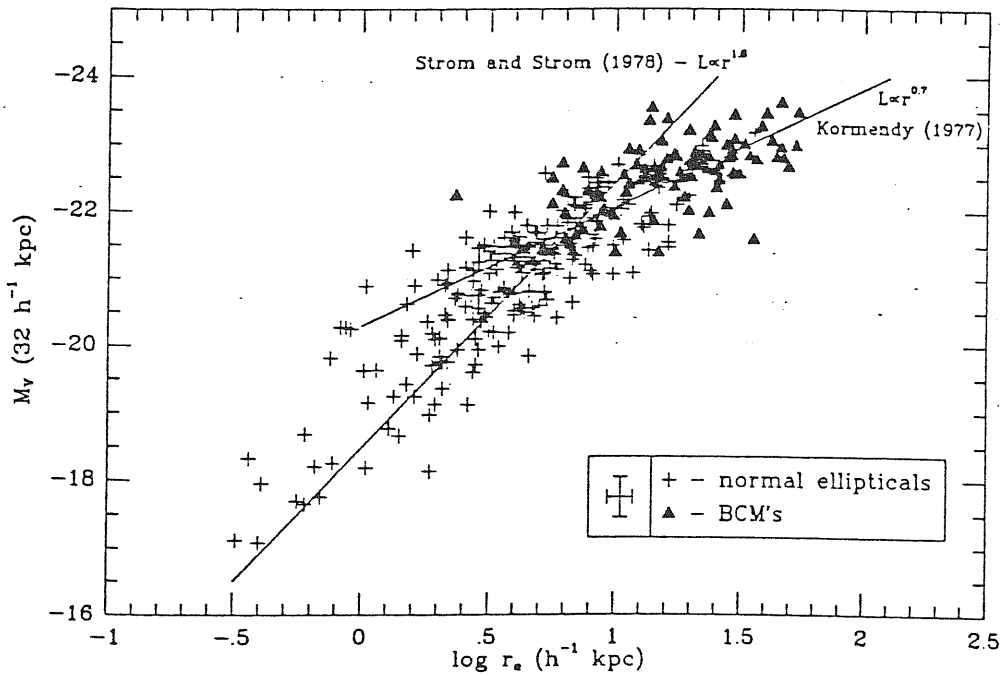

Figure 2.8: Absolute V-magnitude inside the 32 Kpc radius (M_{32kpc}) vs. logarithm of effective radius (r_e) diagram. The two relations from Strom and Strom (1979b) (dominated by faint ellipticals) and Kormendy (1977a) (dominated by bright ellipticals) are also shown. The brightest cluster members (BCMs) are expected to deviate from the relation as a result of their enlarged radii. However, the bright isolated ellipticals also follow the Kormendy (1977a) relation above $M_V = -21.5$. This break is near the same magnitude where Davies et al. (1983) determined that the internal kinematics of ellipticals changes from rotational support to anisotropic velocity support. (from Schombert, 1987)

2.5 The Selection Effects: a Cautionary Tale

The discovery of galaxies is severely limited by observational selection biases. The selection has, however, usually been based initially on photographic material. De Vaucouleurs (1974) has discussed the criteria used in selecting galaxies, and among these surface brightness and apparent diameter. Objects having too low surface brightness are not optically detectable and objects having too small apparent diameter are not readily distinguishable from stars. As a result of these two observational considerations, it is possible to draw a band through the luminosity-diameter diagram (Fig. 2.9), which contains the known galaxies, and outside of which galaxies would not be recognized as such on direct photographs. Since the band is more or less filled, galaxies, that have simply not been discovered could well exist outside this band. Further selection effects arise in the formation of catalogues. The completeness effective limit is mainly a function of surface brightness σ for large objects, of total luminosity σD^2 for smaller objects, and of apparent diameter D only for very small objects which necessarily must have high σ. It is second order function of shape and light concentration (i.e. morphological type) and subject to external factors such as star field density and interstellar extinction. So, it is possible that the trend and the spread of L-D relation is due to the observational capacity *to see* or *not to see* a galaxy with a given surface brightness σ.

From an observational point of view, the galactic disks present a face-on central surface brightness σ_0 of the same value (see section 2.2) within a small range, indipendent of morphological type. This result is known as Freeman's law: $S_0 = 21.65$ B mag arcsec^{-2} (Freeman 1970) (S_0 is the central surface brightness expressed in mag arcsec^{-2}). Similarly Fish's law exist for ellipticals (Fish 1964): $S_0 = 14.80$ B mag arcsec^{-2} (however, more recent and better data show that σ_0 is not the same for all ellipticals, see Kormendy, 1982).

Disney (1976) examined in a more quantitative way these selection effects, in particular those concerning the choice of especially large galaxies for detailed photometry. In his hypothesis it is the apparent radius r_{ap} (and area πr_{ap}^2) which gives a galaxy a spectacular appearance on a plate, and makes us to believe that it is probably luminous and suitable for more detailed investigation. Disney determined the linear diameter r_{ap} at which the image of a disk galaxy of total luminosity L_T is lost in the night-sky noise. Let

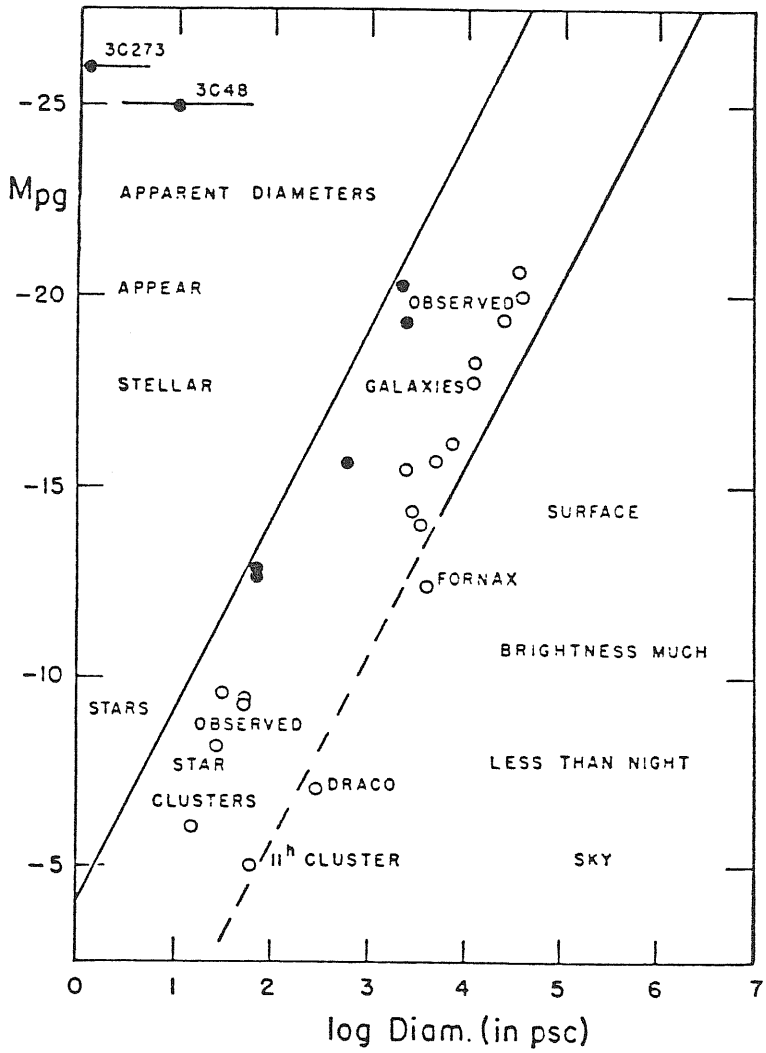

Figure 2.9: Luminosity-diameter diagram showing the region to which discovery of galaxies on direct photographs is limited. Some galaxies discovered by radio emission (quasi-stellar objects) and some that are close enough to notice concentration of individual stars (Fornax and Draco) are shown falling outside this band of observational selection. Line on left is for objects with 1" apparent diameter at apparent magnitude 18.0. Line on right represents a surface brightness of about 25.5 pg mag/square second of arc. (from Arp, 1965)

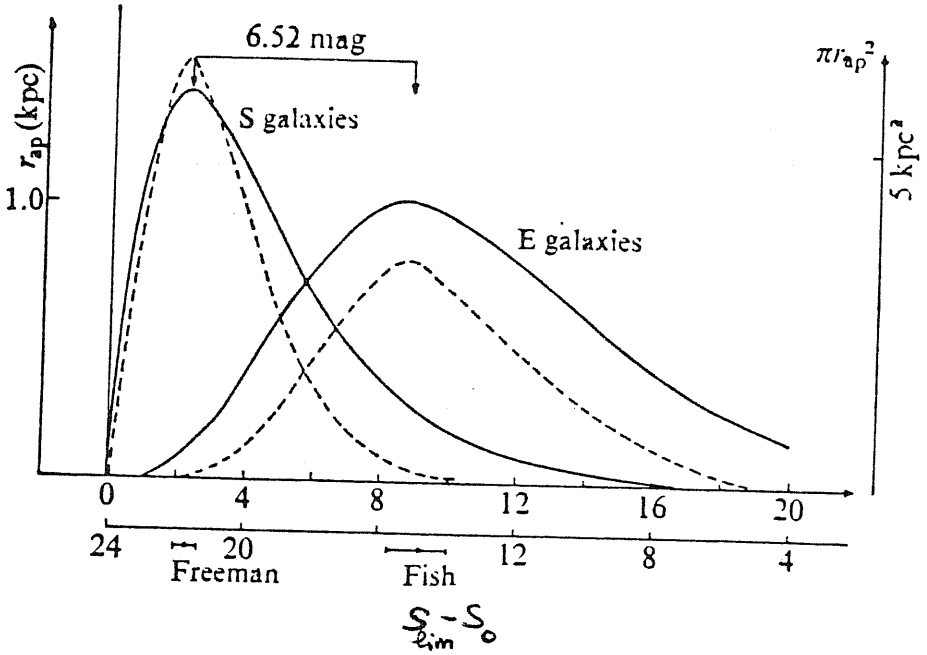

Figure 2.10: Apparent radii r_{ap} (solid lines) and areas πr_{ap}^2 (dashed) for ellipticals and spiral disks of total luminosity $L_{pg} = 10^9 L_{\odot pg}$, as a function of differences between limiting and central B-surface brightness($S_{lim} - S_0$). Radius scale on left, area on right. Changing L varies only the vertical scale. The scale under the abscissa is based on the assumption that $S_{lim} = 24$ B mag arcsec^{-2}. (from Disney, 1976)

the central surface brightness of the system be σ_0 and let σ_{lim} be the limiting surface brightness out to which diffuse objects are readly apparent on a photographic plate, and let S_0 and S_{lim} be σ_0 and σ_{lim} expressed in B-mag arcsec^{-2}. For constant L_T and S_{lim}, one expects that r_{ap} has a maximum as σ_0 varies. If σ_0 is low, even the central isophotes lies close to σ_{lim} and r_{ap} is small; conversely, if σ_0 is high r_{ap} also is small. Using the exponential law Disney (1976) obtained

$$r_{ap} = K\ [L_T/(2\pi)]^{1/2}\ (0.4ln10)\ exp(0.46S_0)\ \delta S \qquad (2.29)$$

where $\delta S = S_{lim} - S_0$ and K is a normalizing constant whose value can be obtained from any well studied galaxy. An analogous formula for r_{ap} is obtained for elliptical galaxies (when the de Vaucouleurs' law for ellipticals is used). In the Fig. 2.10 r_{ap} varies as a function of δS for a constant luminosity $L_{pg} = 10^9 L_{\odot pg}$; the figure shows this dependence for spirals and for elliptical galaxies. These values of δS are such that, if one were to identify the char-

acteristic central surface brightness derived in this way for disk galaxies with the characteristic brightness 21.6 B-mag arcsec^{-2} derived observationally by Freeman (1970) and Schweizer (1976), than one would find S_{lim} =23.8 B-mag arcsec^{-2}, a not unreasonable value for the effective limiting brightness of the conspicuous parts of the images of galaxies. A similar conclusion is reached for ellipticals. Notice, for example, how r_{ap} peaks strongly at a characteristic values of δS, which differ by 6.5 mag arcsec^{-2} for elliptical and disk galaxies: this is in the right direction and very close to the value required to explain both Fish's and Freeman's laws. In conclusion, the selection effect may reveal itself as a tendency to pick up only moderate values of σ_0, and it may cause the observed paucity of galaxies in the too high or too low surface brightness regions in the diameter-luminosity plane.

Allen and Shu (1979) criticized Disney's use of quantities which are not directly observable, such as luminosities instead of apparent magnitudes, and linear sizes instead of angular sizes. Allen's and Shu's analysis considers three possible effects which could enter into the selection of a galaxy for surface photometry: (i) the galaxy must have a small enough total apparent magnitude m to appear in a catalog of bright galaxies; (ii) the galaxy must present a sufficient contrast with the night-sky background, that is a sufficient difference between limiting and central surface brightness $\delta S = S_{lim} - S_0$; (iii) the galaxy must subtend an angle larger than the limiting apparent angular diameter θ_{lim}. The above selection criteria are not independent of one another, since there exists a relation among total apparent magnitude, extrapolated central surface brightness, and apparent angular size (see Allen and Shu, 1979). The final relationship for spirals (a similar formula is presented for ellipticals) is

$$\theta_{lim} = 0.7349\ [(S_{lim} - S_0)10^{0.2(S_0 - m)}]\ arcsec \qquad (2.30)$$

For a given S_{lim},the above equation defines a surface in $(log_{10}\theta_{lim}, S_0, m)$-space; see Fig. 2.11 . Galaxies which satisfy the exponential law (2.5) would lie somewhere on this surface. Galaxies whose light distribution are not well represented by the law will generally be displaced from this surface. Consider now the projection of this surface onto the $(m, log_{10}\theta_{lim})$-plane in the form of a contour diagram for S_0. Fig. 2.12b shows the case when S_{lim} is taken to be 25. The solid line in the figure gives the contour level for the value of S_0 obtained when θ_{lim} is maximized as a function of σ_0, that is $\partial\theta_{lim}/\partial\sigma_0 = 0$, whereas the dashed lines indicate contour levels for S_0 on either side of the

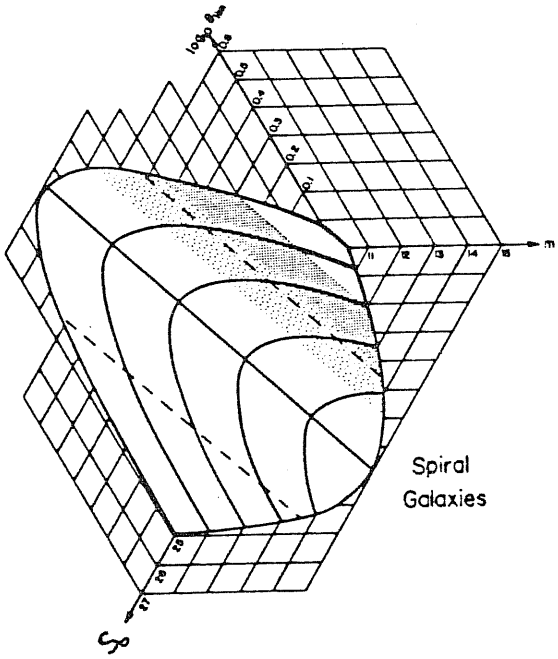

Figure 2.11: The surface occupied by exponential law in ($log_{10}\theta_{lim}$, S_0, m)-space by spiral galaxies. The units of θ_{lim}, S_0, and m are, respectively, arcmin, B-mag arcsec^{-2}, and B magnitudes. (from Allen and Shu, 1979)

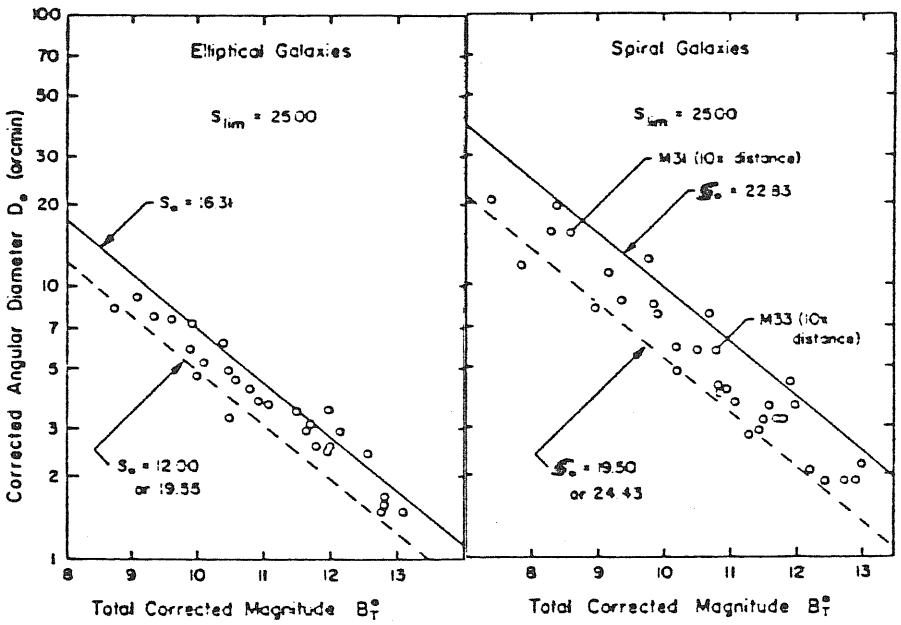

Figure 2.12: Central isophotal contours in the ($m = B_T^0$, $\theta_{lim} = D_0$)-plane for (a) elliptical galaxies and (b) spiral galaxies when S_{lim} =25.00 B-mag arcsec^{-2}. The solid and dashed lines represent the ideal exponential law, whereas the circles give the observed values for Fish and Freeman's samples. (from Allen and Shu, 1979)

value which yields a maximal θ_{lim} for given m. Notice that the region above and to the right of the solid line is forbidden to any galaxy which perfectly satisfies the exponential law, and that both a faint value and a bright value of S_0 are possible for every point in the allowed region below and to the left of the solid line. Similar considerations are possible for ellipticals (Fig. 2.12a). So the authors also agree that there could have been a selection against faint central surface brightness, but they conclude that no bias appears against high surface brightness, that is the observed paucity of galaxies with very high surface brightness.

For early spirals the situation is more complicated: as Kormendy (1977a) pointed out, it is difficult to disentangle the disk and spheroidal components, the latter causing the apparent disk central surface brightness σ_0 to be in error. Phillipps and Disney (1983) also discussed the presence of a bulge.

Disney and Phillipps (1983) demonstrates that the corresponding volume in which the galaxy will be visible (called its 'visibility') is a sensitive function of the galaxy central surface brightness as well as of its absolute magnitude. Their conclusion is that from present catalogues we have rather incomplete knowledge of the population of galaxies , in particular of high and low surface brightness objects. On the base of a sample of spiral galaxies in the Virgo cluster, Phillipps and Disney (1986) concluded that for high luminosities (in the range -20.25$\lesssim M_B \lesssim$ -19), the present data points seem to be constrained well within the sample selection limits. This may suggest a genuinely fairly narrow distribution of central surface brightness. For the lower luminosity galaxies, on the other hand, the galaxies are spread fairly impartially across all the (rather limited) surface brightness range which is accessible (the Fig. 2.13 shows their complexity), so no statement can be made about the true extent of the distribution.

Van der Kruit (1987) performed detailed surface photometry on a sample of disk galaxies with well-defined selection criteria. Since the selection criteria are known in detail he predicted the Disney's selection effects quantitatively using the formulation of Disney and Phillipps (1983) and explicitly corrected his data. The conclusion is that selection as first described by Disney is not responsible for the small spread, in the central surface brightness, he observed and that galaxy disks actually have a remarkably small range in central surface brightness, that is the small spread in luminosity-diameter relation has a physical significance.

An interesting example of empirical study about the influence of selection

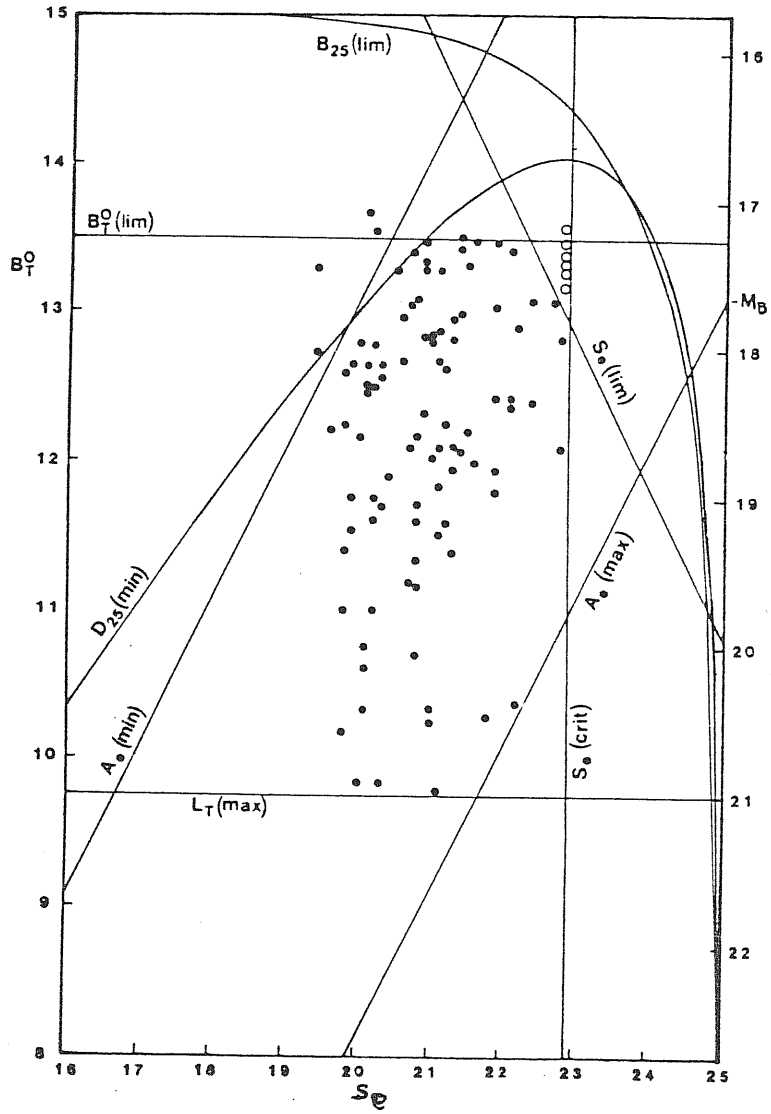

Figure 2.13: Bivariate Brightness Distribution, M_B versus S_e, the 'effective central surface brightness' (see the original paper). The absolute magnitude M_B was calculated using a distance modulus of 30.74 mag. The lines and curves indicate the various selection limits discussed in the original paper (B_T^0 is the corrected total B-magnitude, L_T is the total luminosity, A_e is the effective diameter, D_{25} is the isophotal diameter at 25 B-mag arcsec^{-2} and B_{25} is the isophotal B-magnitude inside the 25 B-mag arcsec^{-2} isophote). The open circles represents galaxies for which S_e could not be properly determined but which are clearly of low surface brightness (see the original paper). (from Disney, 1986)

effects on the L-D relation is presented in the catalogue of Lauberts and Valentijn (ESO). The authors analyzed the selection effects present in their catalogue data on the base of Allen and Shu (1979) and Impey et al. (1988) works. They showed that the face-on magnitude corrected data reproduce the theoretically expected boundary at the lower right side, corrisponding to $S_0 = 22.8$ mag arcsec^{-2}(see Fig. 2.14). On the other hand, they pointed out that no selection effect is expected to produce the sharp cutoff of the sample data at high S_0.

So long, the issue of the selection effects remains for the moment an open question.

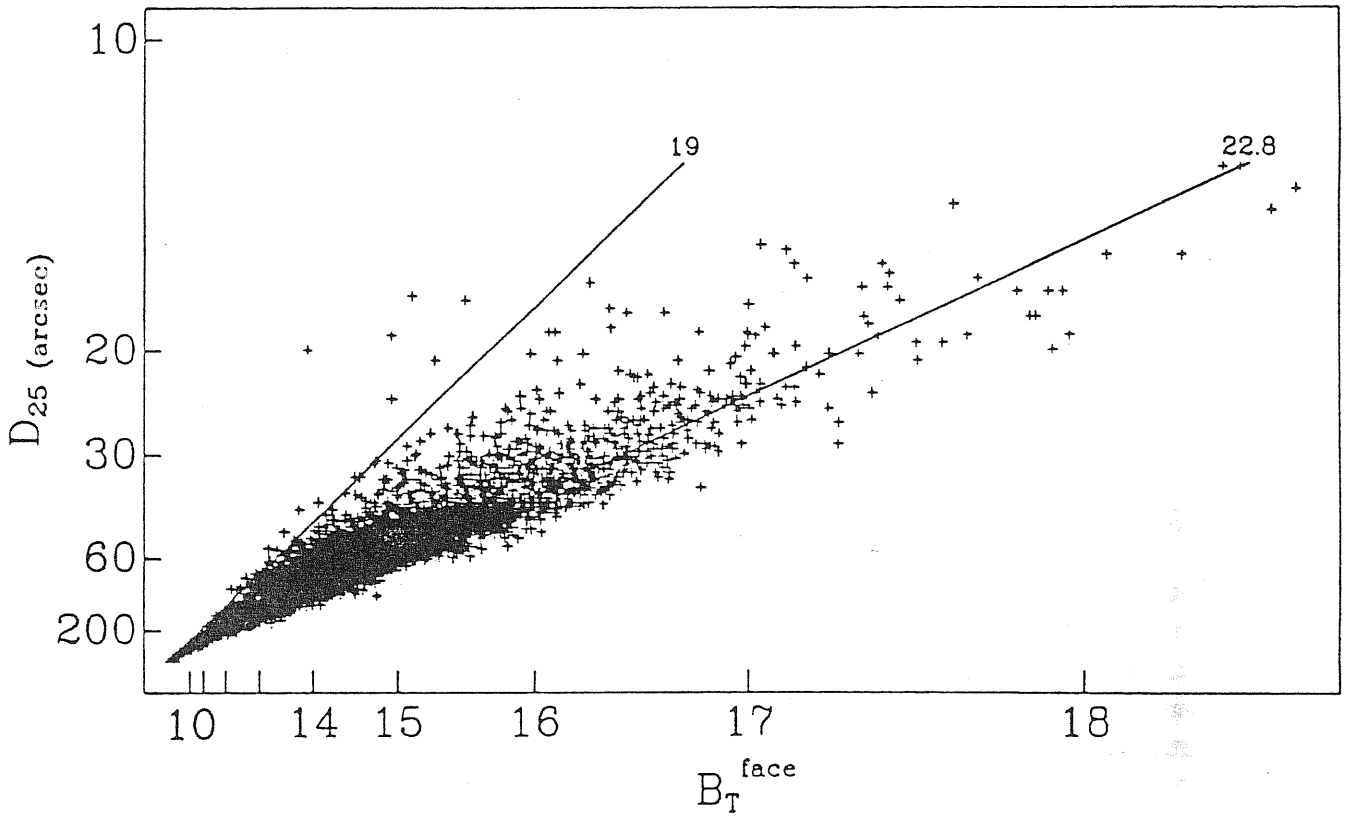

Figure 2.14: Measured data of $\sim$ 9000 late type galaxies plotted; with 'face-on' correction of B_T. Two straight lines of constant central surface brightness are indicated. The 'face-on' corrected data reproduce the theoretical limit at the lower right side quite accurately. (from ESO)

Chapter 3

L-D Relation and the Environment: Some Theoretical Views

Summary

Galaxy formation is very complex. Theories must be compared with observations on disk galaxy properties, and, in particular, with the tight L-D relation; in fact, some theories of galaxy formation explain the general trend of the L-D relation. I discuss the causes of possible differences in the L-D relation for galaxies in different environments. These differences are generally explained by the action of environmental mechanisms. Among these mechanisms there are the truncation and the distension of the galaxies, due to tidal interactions that act on star content and are stronger in denser environments. These tidal interactions may be a local characteristic (galaxy-galaxy interaction) or a global characteristic (galaxy-cluster field interaction), and these interactions may have been stronger in early times. However, the general conclusion is that these mechanisms does not strongly influence the luminosity profiles of most galaxies. In disk galaxies, it is also possible that a decreasing of star-forming activity reduces the the surface brightness and so also the estimated isophotal diameter (at a fixed surface brightness). This truncated star formation may be induced, in the densest environments, by a partial stripping of gas.

3.1 Introduction

Several observations indicate that the environment can affect the structure and dynamics of galaxies (see chapter 1). The role of the environment cannot be easily separated from the galaxy formation process itself. The models that describe the formation and evolution of galaxies fall roughly into three classes (see Dressler, 1984 for a review). In the first class, similar *initial conditions* are assumed for all galaxies, regardless their future destiny as cluster or field galaxies. This is the case, for example, in the hierarchical clustering model (Peebles 1974a,b, 1980) with a perturbation spectrum that is nearly white and randomly phased, because in this model the fluctuations that become galaxies reached the nonlinear growth phase long before the cluster-size perturbations. In this sense, galaxies could not "know" their future environments. The aim is to reproduce all the morphological variation *with fairly late evolution* (after clusters became important); for example, S0s might be produced from stripping and ellipticals from merging of an initial population of spiral galaxies. In the second class, *later evolution* is retained as the primary modifier of galaxy type, in particular through the truncation of disk development, but initial conditions or very early evolution are added to account for the prominence of bulge-dominated galaxies in regions of high galaxy density. This might be accomplished, for example, by dropping the condition of random phasing, so that galaxies with higher central concentration were destined to inhabit regions of high galaxy density (Dressler 1980), or by including mergers in the early evolution of clusters (the small group phase) to build up a population of more massive spheroids. If these types of models are unable to explain the observational data, there is still the third possibility that *initial conditions* were primarily responsible.

Many authors (e.g. White and Rees, 1978; Fall and Efstathiou, 1980; Peebles, 1980; Silk and Norman, 1981; Zurek, Quinn and Salmon 1988) have worked on galaxy formation and have tried to explain how galaxies acquire their principal global properties. There is a large number of correlations between these global parameters, but only two relationships are tight enough to imply that they could be representative for all spirals: the luminosity-velocity relation (Tully and Fisher 1977) and the L-D relation. For istance, as regards the L-D relation, Fall and Efstathiou (1980) proposed that $L \propto D^2$ (for disk galaxies) in comparing their results to observations. As regards disk galaxies, Burstein and Sarazin (1983) derived the luminosity-velocity relation

$L \propto V^p$ (p is a numerical exponent) and the luminosity-luminous radius relations $L \propto R_L^m$ (m is a numerical exponent) from simple physics and a few reasonable assumptions about the relationship of mass to luminosity in spiral galaxies. Both previous relations are derived from a single density-radius power-law relation for spiral galaxies, assuming that the total mass-to-luminosity ratio is fixed by the Hubble type of the spiral, and that spirals gain their angular momentum through tidal interactions (e.g. Peebles 1969, Fall and Efstathiou 1980). Burstein and Sarazin (1983) obtained that $L = A_H V^p$ and $L = B_H R_L^m$ where the quantities A_H and B_H depend only on Hubble type and $m = p/(p-2)$. Using the RC2 magnitudes and diameters Burstein and Sarazin (1983) proposed a value of $m = 1.6 - 1.8$, then p is in the range from $p = 5.3$ to $p = 4.5$. Both values of p are consistent with each of the Tully-Fisher relations derived by Rubin et al. (1982). A detailed discussion of galaxy formation is beyond the aim of this thesis; an interesting result is that some theories can predict the observed trend of L-D relation (see e.g. Fall and Efstathiou, 1980; Burstein and Sarazin, 1983).

As regards the L-D relation in different environments, it must be noticed that there are several environmental mechanism that can affect the structure and the dynamics of galaxies: tidal mechanism, merging, galactic cannibalism, ram pressure stripping and gas evaporation, truncated star formation, cooling flows (see e.g. White 1982 for a review). Some of these previous mechanisms, such as the ram pressure stripping, can change only the gaseus content of galaxies, others, such as merging, can strongly alterate the morphology itself of galaxies. In particular, there is a series of these mechanism that several authors (e.g. Strom and Strom, 1978a-e; Peterson, Strom and Strom, 1979) believe to be able to modify the L-D relation. I summarize the principal points on this topic in the following sections. However, I stress that particular initial conditions may be a viable explanation for possible differences in the L-D relation in different environments.

3.2 Collisional Stripping (Galaxy-Galaxy)

A tidal truncation mechanism arises from the tidal effects of encounters between individual galaxies. This mechanism is clearly of gravitational origin. The effect of the tidal force induces in the galaxies an internal energy gain derived from their relative motion.

These galaxy encounters lead to an injection of energy into the outer layers of each galaxy which is most easily estimated using the impulsive approximation. The approach I use comes from a paper by Spitzer (1958) (see also White, 1982). Consider a star which is situated at a distance $\vec{r}^*$ from the centre of the galaxy and is moving with velocity $\vec{v}^*$ with respect to the centre of its galaxy. Let a perturber of mass m_p pass by rapidly with velocity $\vec{V}_p$ in a relative orbit with pericentre at $\vec{d}$. The star experiences an impulsive velocity change,

$$\Delta\vec{v}(\vec{r}^*) = \int_{-\infty}^{+\infty} dt\ \vec{r}^* \cdot \nabla[-Gm_p(\vec{d} + \vec{V}_p t)/ \mid \vec{d} + \vec{V}_p t \mid^3] =$$

$$= \frac{2Gm_p}{V_p d^2}\ [2(\vec{r}^* \cdot \vec{d})\vec{d}/d^2 + (\vec{r}^* \cdot \vec{V}_p)\vec{V}_p/V_p^2 - \vec{r}^*], \tag{3.1}$$

where $d \gg \mid \vec{r}^* \mid$ and $V_p/d \gg v^*/r^*$ are implicitely assumed (see e.g. White, 1982). The galaxy is thus stretched along the pericentre direction $\vec{d}$, and is compressed in the direction perpendicular to the orbital plane of the encounter. The change in specific energy of the single star is

$$\Delta(1/2v^{*2}) = \vec{v}^* \cdot \Delta\vec{v} + 1/2 \cdot \Delta v^2. \tag{3.2}$$

The energy change thus has two terms. One is first order in the velocity change and may be positive or negative. Over a series of randomly oriented encounters it will cause the star to execute a random walk in energy space. The other is second order and is always positive. The nett effect of a series of encounters on the stellar energy distribution will thus be the sum of a zero mean diffusion process and a secular drift to less bound orbits. The number of encounters, required for the secular term to cause a significant change in the orbit of a star (which initially has specific energy E), is of order

$$N_{sec} = -E/ < \Delta E_{sec} >= -2E/ < \Delta v^2 >, \tag{3.3}$$

where Δv^2 is averaged over the orbit and over all possible encounters. Similarly the number of encounters required for diffusion to change the orbit significantly is (see e.g. White, 1982)

$$N_{diff} = E^2/ < \Delta E_{diff}^2 >\simeq 3E^2/ < v^{*2} >< \Delta v^2 >, \tag{3.4}$$

where v^{*2} is averaged over the orbit and correlations between $\vec{v}^*$ and r^* have been neglected. Since we expect $E \simeq -1/2 \cdot < v^{*2} >$, we see that the first and second order terms (in eq.3.2) cause orbital evolution on the same timescale (Knoblock 1978). Even in a single encounter, however, the total energy change in a galaxy usually comes from the second order term alone; the first order term averages to zero both in a system without streaming motions and in an axisymmetric rotating system. The mean specific energy change of stars in a shell of radius r^* is easily shown to be (Spitzer, 1958)

$$\Delta E = 4/3 \cdot G^2 m_p^2 r^{*2}/d^4 V_p^2, \tag{3.5}$$

when the first order term (in eq.3.2) averages to zero. If we consider shells in the outer parts of the galaxy we may estimate their specific binding energy by $E \simeq -Gm(r^*)/2r^*$ and so obtain,

$$\Delta E/E = -\frac{8}{3} \cdot \frac{m_p/d^3}{m(r^*)/r^{*3}} \frac{Gm_p}{dV_p^2}. \tag{3.6}$$

If we define a truncation radius by the value $\Delta E/E = -1$ we obtain almost the Roche criterion (see equation 3.9 in the following section). Notice that although energy changes in a single collision only affect stars at the Roche radius rather weakly, repeated collisions may have significant effects at considerably smaller radii where the rapidity of the encounters makes adiabatic invariants ineffective. According White (1982), the above derivation of a truncation radius is suspect because it uses only the second order secular term in (3.2). It is clear, however, that the first order term can also induce particles to move to positive energy, and may even be dominant if many stars have $\vec{v}^*$ nearly parallel to $\Delta\vec{v}$ and $v^*/v_e \sim 1$, where v_e is the local escape velocity modulus. Experiments carried out so far suggest that this may occur for galaxies with near radial stellar orbits in their outer regions. But, in other cases, resonance and capture effects (which cannot be treated in the impulsive approximation) are a major cause of escape for encounters at realistic velocities (Richstone 1975, Dekel et al. 1980, Gerhard 1981).

When the first order term in (3.2) averages to zero, the change in total binding energy of the galaxy is simply the integral of the second order energy change over the entire system minus the kinetic energy at infinity of the escaping stars. Since in numerical experiments the velocities of escaping stars are generally found to be small compared to the internal velocities of

their parent galaxies, this last term can be neglected and we can write (see e.g. White, 1982)

$$\Delta E_{tot} = 4/3 \cdot G^2 m_p^2 < r^{*2} > / d^4 V_p^2, \tag{3.7}$$

where the rms radius of stars in the initial galaxy is mass weighted. If we use the definitions $E_{tot} = -Gm/4r_g = -1/2 \ < v^{*2} >$ for the gravitational radius and rms velocity of the galaxy, (3.7) can be rewritten as

$$\Delta E_{tot}/E_{tot} = -\frac{32}{3} \cdot \frac{m_p}{m} \frac{< v^{*2} >}{V_p^2} \frac{r_g^2 < r^{*2} >}{d^4}. \tag{3.8}$$

This formula shows firstly that in an environment as a galaxy cluster, most of the tidal effects on a galaxy will be the result of the few closest encounters that it has suffered, secondly that slow encounters are considerably more effective than rapid encounters in causing changes in structure, and, finally, that the most massive perturbers have the greatest influence on galaxy evolution. These results have an unfortunate consequence. The most important encounters for the evolution of galaxies are precisely those interpenetrating, relatively slow encounters for which the impulsive approximation and the chain of assumptions leading to (3.8) are all invalid.

Several authors worked on these topics. I hint only to some of them. The calculations of Gallagher and Ostriker (1972) led to an estimate of less than 1 per cent for the amount of luminosity collisionaly removed in a Hubble time from a typical elliptical galaxy. This result suggests that tidal encounters will not have any strong influence on the luminosity profiles of galaxies in rich clusters. These calculations were carried out before massive halos became fashionable, and one might wonder whether adding such *halos* might make stripping more efficient. Richstone (1975) simulated a number of rapid collisions between galaxies allowing explicitly for the extended mass distribution of the galaxies. Richstone adopted King's (1966) model as a galaxy model. King models have the virtue of fitting reasonably well the observed light distributions of certain ellipticals; moreover, it may also be a not too bad approximation for spirals since there is observational evidence that spirals have a halo. Richstone (1976) concluded that if galaxies in a rich cluster had originally possessed very extensive massive halos (100 Kpc or more), collisions would have by now imposed a tidal limit of about 50 kpc on those galaxies which spend most of their time at the cluster centre; moreover the minimum

truncation radius they find is still larger than the directly observable extent of galaxies in rich clusters. This estimate is confirmed by Merritt(1983). Their results thus agree with those of Gallagher and Ostriker in suggesting that the luminosity profiles of elliptical galaxies are unlikely to be modified significantly by encounters. In fact, by generalizing the impulsive theory, it is easy to show that the effect of encounters on the visible parts of some test object should depend only weakly on the fact that its perturbers possess massive halos (see White 1982).

3.3 Stripping from the Mean Cluster Field

Tidal stripping by collisions is not the only stripping mechanism which operates in rich clusters. Peebles (1970) and Gunn (1977) have pointed out that the tidal field of the cluster potential well (that is the tidal field determined by the total mass density, irrespective of how it is divided among galaxies) can be quite effective in removing matter from galaxy halos. This process is exactly analogous to the truncation of globular clusters by the tidal field of the Galaxy and will henceforth be called *tidal limitation.* It is a process distinct from *collisional stripping* (see the previous section), which results from the impulsive tidal field produced by the near passage of two galaxies. The relative importance of the two processes obviously depends on the fraction of cluster mass that resides in galaxies, since the mean tidal field is fixed by the depth of the cluster potential well, while the strength of a collision depends on the masses of the colliding galaxies. Furthermore, whereas collisions act gradually to reduce the mass and radius of a galaxy, tidal limitation occurs on roughly in an orbital time scale and it is probably effective as soon as a cluster has formed (see e.g. Merritt, 1984).

As a galaxy orbits about its companion or through its cluster it is subject to a time-dependent tidal field that can have significant effects on its outer regions. The effect of this field is most easily described using the classical Roche theory. Consider a galaxy with mass profile $m(r)$ in a circular orbit of radius R_0 about a cluster with mass profile $M(R)$. Then equating the differential cluster force on a star distance r from the centre of the galaxy to the restoring force due to the galaxy itself leads to a tidal radius r_t given by (see e.g. White, 1982)

$$m(r_t)r_t^{-3} = (2 - dlnM/dlnR)M(R_0)R_0^{-3} \tag{3.9}$$

Note that this tidal radius is similar to that of equation (3.6), except that in (3.6) tidal effects have been weakened by the factor $4/3 \cdot Gm_p/dV_p^2$ which accounts for the fact that the perturber is moving much faster relative to the galaxy than it should if galaxy and perturber were on a circular orbit about each other.

In the equation (3.9) the tidal cut-off is thus imposed at the radius at which the mean density of the galaxy becomes equal to the mean density of the part of the cluster contained within its orbit. This statement is equivalent to say that the galaxy is limited at the radius at which the orbital period of a star in the galaxy becomes equal to the orbital period of the galaxy about the cluster. Stars which are well within this radius see a slowly varying tidal field, and their orbital characteristics are protected from long term evolution; stars which are outside this radius pursue an orbit about the cluster center which is almost independent of that of the core of their galaxy, and are thus separated from it by phase mixing. This kind of process appears to limit the radii of globular clusters in our own Galaxy and of dwarf galaxies orbiting large members of the Local Group (King 1962). For non-circular orbits it is usually assumed that R_0 should be set equal to R_p, the pericentre of the orbit. If we adopt this assumption and model the galaxy and the cluster as isothermal spheres of one-dimensional velocity dispersion σ_g and σ_c respectively (see e.g. Merritt, 1984), then (3.9) becomes

$$r_t/R_p = \sigma_g/\sigma_c \tag{3.10}$$

so that in a rich cluster a galaxy is truncated at a radius equal to about one fifth of its distance of closest approach to the cluster centre. Since a typical galaxy has a pericentre about a factor of two smaller than its current distance from the centre, the mean cluster field is effective at truncating any massive halos that cluster members may possess. It is unlikely, however, to have any noticeable effect on the visible parts of galaxies. Note that the arguments on which (3.9) and (3.10) are based are very crude, so that the derived tidal radii are likely to be good only to within a factor of two or three. Keenan (1981) studies the problem of tidal truncation in more detail using restricted 3-body methods; he recommends that the tidal radii obtained above should be decreased by a factor of 1.6.

A comparison of equation (3.10) with the results of Merritt (1983), which incorporate Richstone's (1975, 1976) stripping cross-section into a self-consistent tratment of the evolution of a galaxy cluster, suggests that the mean field of the cluster may be significantly more efficient than collisions in removing the outer parts of galaxies. The effects of the cluster's tidal field were examined by Merritt (1984). The author obtained that tidal radii are of the order of 15 kpc (with H_0= 100 km s^{-1} Mpc^{-1}) for galaxies near a cluster core. Merritt pointed out that tidal limitatition would not be expected to have a strongly observable effect on most cluster galaxies. Merritt claimed also that removal of matter from galaxy halos by galaxy-galaxy collisions is probably of negligible importance in virialized clusters. Moreover, the stripped material would probably end up in the extended halo of the central cD galaxies.

3.4 Effects on Observed Disks

Numerical simulations have typically some difficulty to deal with the cold disk of spirals and so the works here presented are based on spherically simmetric galaxies, but, as Richstone (1975) has pointed out in his paper [1], the general results may not be too bad. The previous discussion indicates that the above mentioned processes are unlikely to have much effect on observed disks. However, tidal encounters in clusters might, in principle, be able not only to truncate the outer parts of galaxies, but also to rearrange their internal structure, for example by disrupting or fattening the disks of spirals and S0 galaxies.

Richstone (1976) examined the possibility that S0 galaxies be fattened sufficiently by collisions so that they may be identified as ellipticals galaxies. Marchant and Shapiro (1977) claimed that a significant number of elliptical galaxies may be "transformed" S0s and spirals whose disks have been puffed up by encounters. This proposal always looked unlikely and it has been shown to be quite untenable from both direct simulations (Farouki and Shapiro, 1981) and by a careful consideration of the impulsive energy input into a disk galaxy (Gerhard and Fall, 1982). Marchant and Shapiro were misled by their overly simplistic representation of galaxies by homogeneus spheroids, and it seems that encounters in rich clusters will at most thicken

[1] since also spirals seem to have spherical massive halos (see e.g. Roberts and Rots, 1973)

and truncate the outer edges of galaxy disks (and also this effect seems to be limited: see previous sections). Palmer and Papaloizou (1982) have carried out calculations of the effect of slow encounters on a rotating disk: they claimed that energy and angular momentum changes have either sign depending on the parameters of the encounter and the structure of the disk.

3.5 Tidal Encounters at an Early Time

In view of the above discussion about tidal effects, it is very surprisig that there is observational evidence for strong tidal effects in the luminosity profiles of galaxies in some nearby rich clusters (see e.g. Strom and Strom, 1978a-d for ellipticals, and Peterson, Strom and Strom, 1979a for spirals, section 4.1). However, a possible explanation for these results might be that tidal radii of galaxies were not established by their present environment but by their past environment. So, it is possible that most of the eventual tidal damage done to any particular object was inflicted in an environment which differs considerably from the one which currently surrounds it. For instance, as we have previously seen in a previous section (eq.3.8), the tidal effects on a galaxy are stronger if the collision velocity (V_p in eq.3.8) is lower (in a cluster the velocity dispersion is a good estimate of the collision velocity between two galaxies). The velocity dispersion in observed clusters is very large (of the order of 10^3 km/sec), and so the tidal effects are limited. But, assuming a hierarchically clustering universe, in an early time galaxies may have inhabitated dense, low velocity dispersion clumps which may have preceded observed rich clusters, and in these clumps the tidal effects may have been stronger (see e.g. White, 1982). Merritt (1983, 1984) argued that the tidal effects may be so strong during the initial cluster collapse that subsequent evolution of the galaxies may be negligible.

3.6 Tidal Distension and Other Tidal Effects

As previus reported, the more widely known effect of tides is to truncate the halo (*tidal truncation*): this seems to occur when the encounters are more violent, i.e., when the victim is much less massive than the perturber, or when it is located near the center of a rich cluster. However, there are some observational indications that mild encounters produce a distension of the

victim's envelope (Kormendy, 1977c, 1980). This process (*tidal ditension*) was interpreted by Kormendy (1977c) as a combination of reversible tidal stretching and the above mentioned tidal heating.

Moreover, Miller (1988) has argued that a severe "shaking" of a galaxy when it passes by galaxies in the high density core of a cluster may rearrange the distribution of mass into a more elliptical-like distribution without actually adding or removing mass.

3.7 Tidal Truncation or Distension? : A Comparison with Observations.

It seems important to decide whether tidal encounters produce truncation or distension, since distension could increase encounter cross sections and lead to a runaway process. Aguilar and White (1986) used N-body simulations to study the time evolution and the final shape of the density profiles of galaxies that have undergone a tidal encounter. They considered nonrotating spheroids with density profiles given by de Vaucouleurs, and they predicted their final profile. The authors showed how observational parameters such as isophotal radii change as a result of tidal encounters.

Aguilar and White concluded that "tidally distended" objects may be the consequence of a transient phenomenon produced by the radial mixing of stars onto less bound orbits: they suggested that the elliptical galaxies, found by Kormendy (1977c,1979,1982), which show a luminosity excess in their outer regions, may be interpreted as the previous objects. In general, they concluded that strong collisions produce a smaller isophotal radius while weak collisions have the opposite effect. However, the increase in the isophotal radius is always very small, the maximum corresponding to a 3% increase. Most collisions produce a small effect (5% or less), and most of them produce a decrease in the isophotal radius (see Fig. 3.1). They have estimated the magnitude of this effect for a galaxy in the core of a rich cluster and found it to be somewhat smaller in the mean than the effect observed by Strom and Strom (1978a-d). In addition, they noted that strong collisions result in a higher characteristic surface density, while weak collisions produce the opposite effect.

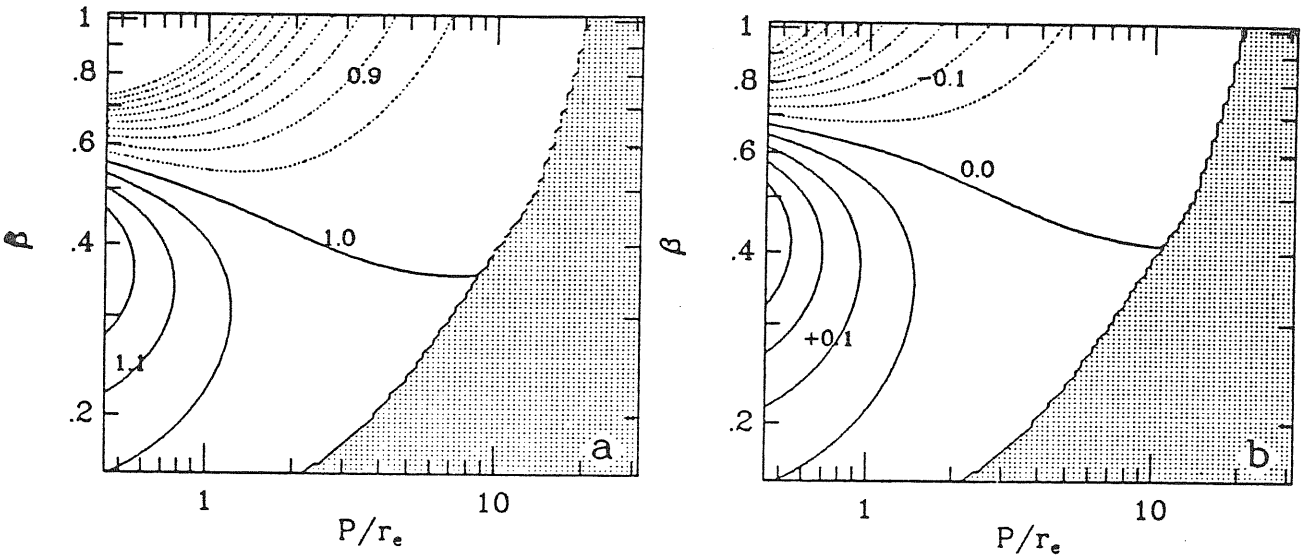

Figure 3.1: Lines of constant ratio of final to initial r_e (effective radius) and of constant change in μ_e (effective brightness magnitude) are presented as a function of impact parameter P and collision strenght β (see the original paper for details) in figures a) and b), respectively. Solid and dotted contour lines correspond to expansion (or dimming) and contraction (or brightening), respectively. The contour lines are at 0.05 intervals in a) and at 0,05 mag intervals in b). The hatched area to the right is the region where changes in mass and binding energy are less than 3%; the formulae used in computing the contour lines are not valid for such weak collisions.

3.8 Truncated Star Formation

There is a long series of mechanisms which explains how the gas content of a galaxy can be removed (see White, 1982). Consider a galaxy at rest in a hot (at a temperature of 10^5-10^8 K) and uniform gas: there is a mass lost for evaporation (see White 1982). Similarly, when a galaxy moves through a gaseous medium, ram pressure effects exist. From these and other effects, it appears that galaxies embedded in the hot intergalactic medium, found in the central regions of rich galaxy clusters, will be stripped of gas. Effectively, as I have already hint in section 1.4, there is some observational evidence that the hydrogen content of clusters galaxies is lower than that typical of a sample of field galaxies (see section 1.4). The gas stripping may affect the formation of galaxy disks.

Larson (1972a,b) has argued that infall from low-density, gaseous halos (possibly remnants from protogalactic condensations with size of several hundred of kpc) significantly affects disk-system evolution over a large fraction of a Hubble time. If such extensive halos are common, it is conceivable that the outer regions of disk systems were formed during the last several billion years. Halos surrounding disk galaxies located in the central regions may be removed; some disk systems in the outer-cluster region might be unaffected owing to the lower density of intergalactic gas and of other galaxies. It was suggested by Strom and Strom (1978e) that the sizes of S0 disks are smaller in the densest regions because the growth of disks is truncated early in dense regions. Peterson, Strom and Strom (1979a) also suggested that the partial stripping of gas could induce the decreasing of star-forming activity in dense-environments galaxies. So the B-surface brightness could be lower for galaxies in denser environments and the possible result is a smaller estimated isophotal diameter at fixed B-surface brightness.

Larson,Tinsley, and Caldwell (1980) have suggested that the lack of spiral galaxies near the centers of clusters might be caused by the removal of these gaseous halos during cluster collapse. Fall (1983) questioned the previous hypothesis; he based his conclusion on considerations about angular momentum. In short, the slow rotation of the ellipticals implies the removal of so much material that it becomes difficult to understand why ellipticals are more massive than spirals.

However, the ram pressure of intergalactic gas acting on the interstellar medium of a galaxy would have been more effective in stripping galaxies

in small clusters at early times than in typical present-day objects (White 1982).

Chapter 4

Environment and L-D Relation: a New Result on Disk Galaxies

Summary

Several authors discussed the possibility that the L-D relation may be different for galaxies in different environments. So long, there is no agreement about the question for disk galaxies. This fact and a recent paper about L-D relation for ellipticals (Giuricin et al., 1989) induced me to reconsider the L-D relation for disk galaxies. I have collected several samples of disk galaxies, in order to study, in each of them, the relation between the blue total corrected absolute magnitude and the absolute isophotal diameter. These L-D relations have been compared to detect a possible dependence on the galaxy environment. No significant differences have been found among the several relations, especially if selection criteria relative to the various samples are taken into account. This result is in disagreement with several previous claims. My result can be restated by saying that the environment has a negligible effect on L-D relations, either because these are weakly affected by the local density, or because the scatter in the relations is too large and the accuracy in the data is too low to allow me to detect any significant deviation from sample to sample.

4.1 The L-D Relation in different environments

The possibility that the L-D relation differs for galaxies in different environmental settings was discussed by several authors. In this section there is a brief summary of inherent papers: for the sake of completeness I report not only results for disk galaxies, but for elliptical galaxies too.

Gudehus (1973) used his and other authors' photometric data (see the original paper) to extend the information on the radius versus magnitude relation to three different environments: three clusters (Abell 754, 1367, 2065) of different classification. He showed that the mean radius-magnitude relation may differs significantly among these clusters: however the data consdered are limited to E-S0 galaxies. Moreover the related sizes of the radius-parameters do not appear to be well associated with Abell's classification (connected with the average density, see, e.g., Bahcall 1977).

In a series of papers Strom and Strom (1978a,b,c,d) presented an extensive photometric survey of elliptical members of several clusters and reported their conclusions about the dependence of elliptical galaxy diameters, at fixed luminosity, on their environmental settings. In the first paper (Strom and Strom, 1978a) the authors noted a difference in mean effective radii between their Coma sample and the sample of E galaxies (primarly in lower-density regions) of Kormendy (1977c): the Coma ellipticals turned out to have slightly larger effective radii at a given luminosity. In the same paper (1978a) they found a difference (that is in some way at variance with the previous one) in mean effective radii between the E galaxies locating in the dense central regions of the Coma cluster compared with those in the outer parts of the cluster: the sample of outer-region galaxies does not appear to contain "concentrated" systems with small effective radius. Summarizing, Strom and Strom (1978a,b,c,d) studied the Absolute Magnitude-Effective Radius (and Isophotal Radius R_{26} [1]) relationship for elliptical galaxies in very different clusters: two dense spiral-poor clusters (Coma and Perseus), one relatively dense, spiral rich aggregate (Abell 1367), a dense, spiral-poor, cD cluster (Abell 2199) and two relatively low-density clusters (Abell 1228 and Hercules). The authors (Strom and Strom, 1978d) concluded that: i) there is a tendency for ellipticals to be smaller (at a fixed luminosity) in

[1] the radius at the 26 mag arcsec^{-2} surface brightness

denser clusters , SP (spiral-poor) and cD clusters, ii) a similar tendency towards small radii (at fixed luminosity) holds for ellipticals located near the central, highest-density regions of the SP/cD compared to those located in the outer regions of these clusters. There is some evidence (Strom and Strom, 1978e) that even S0 galaxies in the outer part of the Coma cluster appear to have larger isophotal diameters than their counterparts in the inner regions of the cluster.

One year after, in 1979, Peterson, Strom and Strom published a paper on disk galaxies, but this study is not so complete and extensive as the previous one on ellipticals. The authors presented photometric data for 32 disk galaxies in two, relatively low-density, spiral-rich clusters, Virgo and Hercules. Peterson, Strom and Strom found that the mean isophotal radius, (corresponding to a surface brightness of 26.6 mag arcsec^{-2}) of the disk systems in Virgo is smaller by 30% compared with those in the Hercules cluster (see Fig. 4.1). Moreover, the authors noted that the typical sizes of the Virgo disk systems appear to be smaller by a factor 1.3 than that for the field galaxies in Holmberg's sample (Holmberg, 1975).

As regards the disk galaxies, Bosma (1985) found an environmental dependence of the relation between the 21-cm line width and the infrared brightness of spiral galaxies and suggested that this effect could be explained by differences in the diameter of galaxies belonging to clusters and fields.

In order to cast some light on galaxy properties in fairly low-density environments, Giuricin, Mardirossian and Mezzetti (1985) collected (and sometimes reduced) corrected isophotal diameters (at 25 B-mag arcsec^{-2}) and corrected blue total magnitude for galaxies belonging to Geller and Huchra's (1983) groups; these groups was successively divided in subsamples according to a compactness parameter. The authors found that the L-D relations , relative to groups of different degree of compactness, do not differ significantly from each other in the case of ellipticals and lenticulars. However, spirals located in groups of high compactness exhibit a L-D relation flatter than the relation characteristic of spirals of low-compactness groups.

In a more recent paper Valder (1986) found that, at a given luminosity, the Virgo ellipticals are more compact than Coma ellipticals.

Giuricin, Mardirossian and Mezzetti (1988) collected (and sometime reduced) corrected isophotal diameters (at the 25 B-mag arcsec^{-2}) and corrected blue total magnitude for roughly 500 lenticular and spiral members of several clusters (Virgo, Hercules, Pegasus I, Abell 262, Abell 1367, Abell

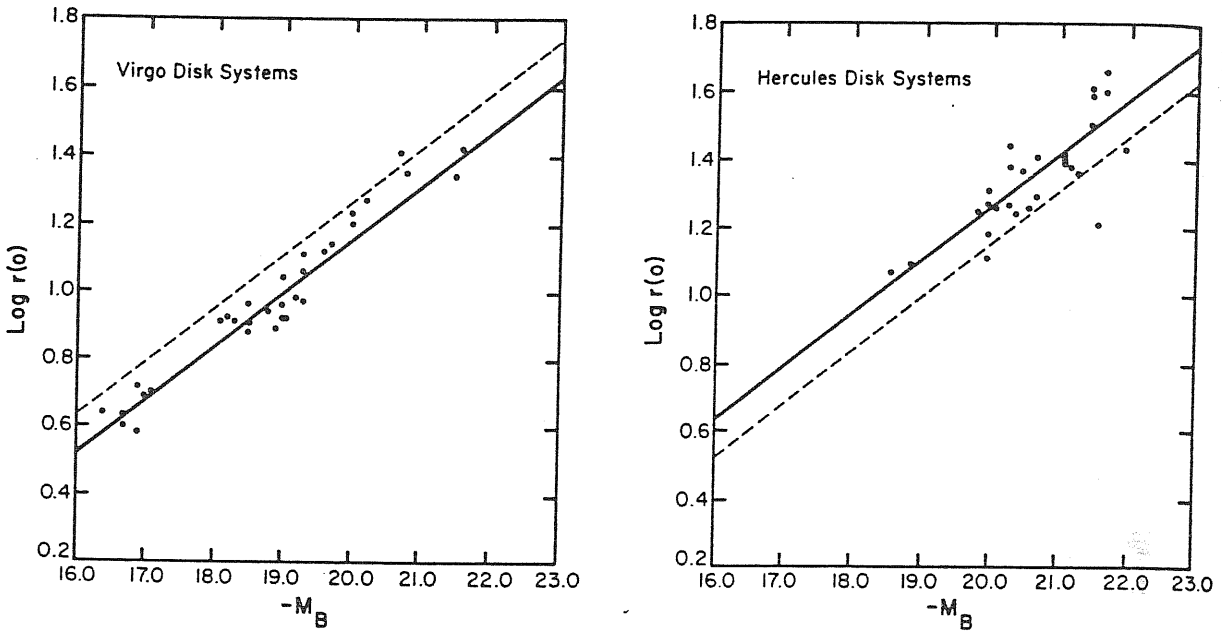

Figure 4.1: a) The radius-luminosiy relation (at the 26.6 mag arcsec^{-2} isophote). The logarithm of the face-on radius $r(0)$, in kpc, is plotted against absolute B magnitude. The solid line represents the linear least-squares fit to the Virgo data, while the dashed line represents the best-fit ($log\ r(0)$, M_B) relation for the Hercules cluster. b) Same as a) except for the Hercules cluster. The solid line represents the linear least-squares fit to the Hercules data, while the dashed line represents the best-fit ($log\ r(0)$, M_B) relation for Virgo. (from Peterson, Strom and Strom, 1979)

1656, Zwicky 74-23): the aim of the authors was to consider a data sample for disk galaxies larger than those discussed in the previous literature. These authors evidenced appreciable differences in the L-D relations for disk galaxies belonging to different clusters, but the character of these relations does not appear to depend on the presently observed main properties of clusters.

In a more recent work, Giuricin et al. (1989) used the homogeneus and extensive survey of photometric data of Burstein et al. (1987) to rediscuss the L-D relation of roughly 160 elliptical galaxies belonging to different clusters (Virgo, Coma, Perseus, Fornax, Abell 2199, Dressler 2345-28), to pairs identified by White et al. (1983) and to the groups identified by Geller and Huchra's (1983), see Fig. 4.2 . The authors found no significant changes in the L-D relations of ellipticals located in the different above mentioned regions; they also examined the L-D relations for inner and outer shells in several clusters. They considered three different types of diameters: corrected isophotal diameters (at 25 B-mag arcsec^{-2}), the effective diameter, and the isophotal diameter corresponding to a corrected mean surface brightness of 20.75 B-mag arcsec^{-2}.

The result of this last paper has prompted me to reconsider the L-D relation for disk galaxies and its possible dependence on enviromnents.

4.2 The Choice of Data Samples

I collected several data samples from the literature. I looked for two basic observational parameters, the isophotal diameter D_{25} and the total blue magnitude B_T, which are the most used in the literature. In the choice of data samples, many factors have been evalutated; in this section the principal ones are considered.

As reported in section 2.1, some *corrections* must be applied to the observed magnitudes and diameters, so I corrected these quantities both for the galaxy's internal extinction and our own galaxy's extinction, and (the magnitudes) for K-dimming. When only the uncorrected data were available, I followed the "Second Reference Catalogue of Bright Galaxies" by de Vaucouleurs, de Vaucouleurs and Corwin (1976; RC2), in the application of the necessary corrections. The correction factors are so important that I discarded, from my samples, any galaxy whose magnitude and/or diameter was neither corrected in the literature, nor correctable, for lack of some of

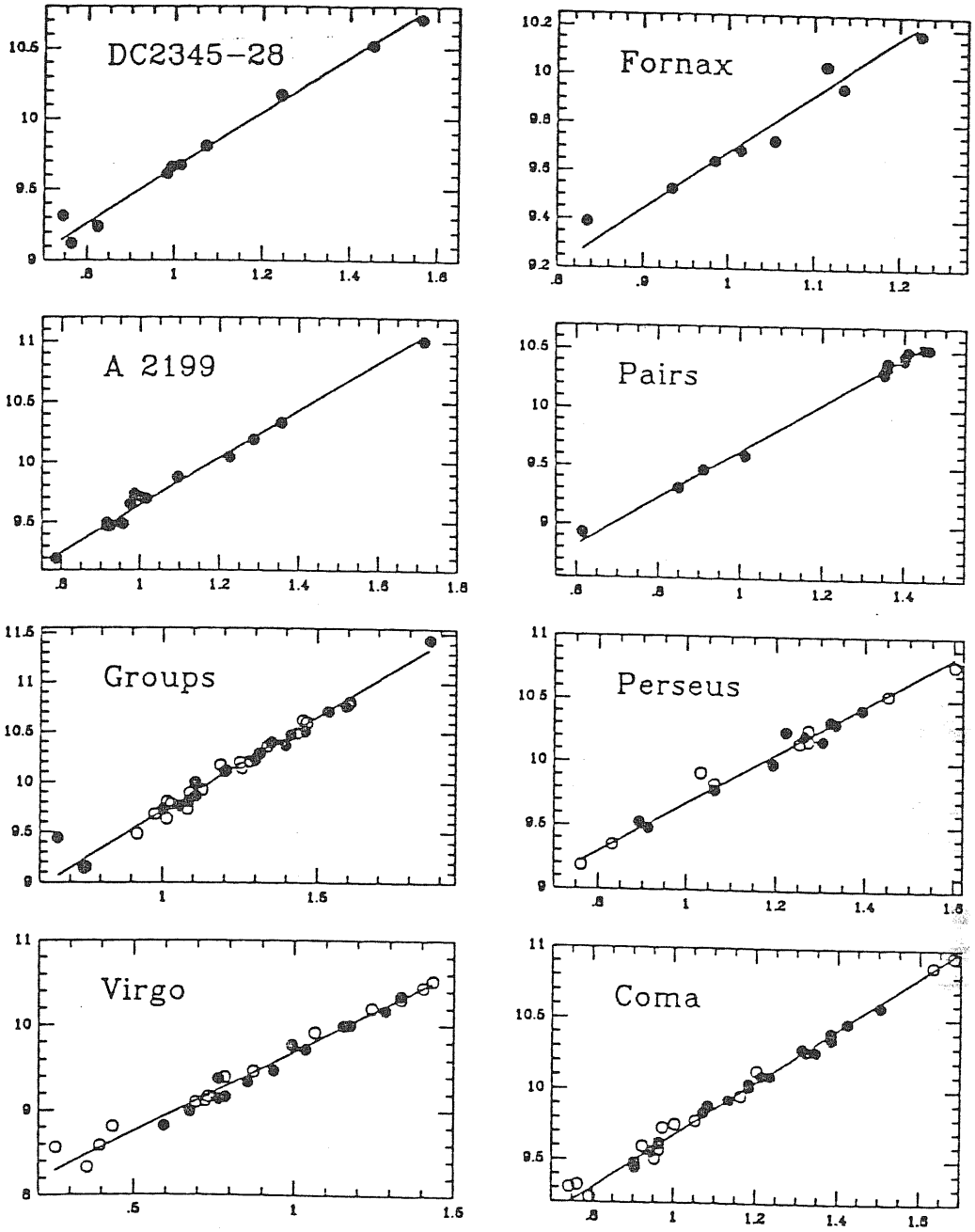

Figure 4.2: Plots of Log L (L is the corrected total blue luminosity in solar units) versus log D (the isophotal diameter D_0 at the 25 B-mag arcsec^{-2} brightness level is in kpc) for the various sets of elliptical galaxies. Inner and outer galaxy member of clusters are denoted by open circles and dots, respectively. The galaxies in groups of high and low compactness are detoted by dots and open circles, respectively. The derived regression lines are also shown. (from Giuricin et al., 1989)

the necessary parameters (axial ratio and radial velocity).

Moreover other parameters are necessary in order to include a galaxy in the data samples: the morphological parameter and its distance. A *morphological parameter* was needed to select only disk galaxies, ranging from S0 lenticulars to Irregulars.

Then the *distance* of the galaxy is necessary to obtain absolute quantities (see section 2.1), but the choice of a distance parameter is often problematic (see e.g. Rowan-Robinson 1985). The following choices were adopted. The cluster galaxies were given the mean Hubble distance of their own cluster (I adopted the membership assignments as given in the sources of the data). The mean heliocentric radial velocity of each cluster was taken from the literature and corrected for the peculiar motion of the Local Group as in Chapman, Geller and Huchra (1988). If the galaxy was part of a "field" or "group" sample and its distance was not given among the other data, I estimated it via the Tully-Fisher distance estimator (Tully and Fisher 1977), with the knowledge of the galaxy's 21-cm line-width. The Tully-Fisher estimator was used because, in the adopted data samples, the galaxies are so close that the distance corresponding to the unperturbed Hubble flow is quite uncertain (because of peculiar motions)

In order to have a good statistics, it is important to work with *large data samples*, so I considered samples with at least ten galaxies, but, in general, much larger.

It is also preferable to compare *homogeneus data samples*, that is data samples with the same photometric source, the same corrections and even the same criterium of aggregation (as regards distinction between cluster, groups and field). Contrarily to the case of ellipticals (Giuricin et al., 1989), I have not found an extensive survey of homogeneus photometric data. However, there are three points which confirm the validity of my results even if the data sample, in its complex, is not homogeneus: i) I have examined data from data samples by different authors and I have tested their *consistency* (see after), ii) some of my comparisons are made among homogeneous samples and the results of these comparisons are in agreement with the general conclusions of the study, iii) if some differences are present among the compared data samples, these differences might induce a spurious difference among the obtained L-D relations, thus increasing possible real differences.

Then, it is fundamental to consider data samples that span a *very large range of environments.* The adopted data samples provide the necessary

variety of environments: I have treated clusters of different density, groups, field. Moreover, I obtained a more subtle environmental division with a parameter of density.

In conclusion, I think to have collected well-reduced data in a large sample that span a very large variety of environments. In the following part of this section I present a brief description of data samples used.

Bothun et al. (1985) and Aaronson et al. (1986) provided a homogeneous data set for disk galaxies in 10 clusters; I retained only 7 of them, since the cluster nature of Cancer and Z74-23 has been questioned (Bothun et al. 1983, Bothun et al. 1985, Aaronson et al. 1986), and Abell 2634/66 had not enough data available. The remaining clusters are: (1) Pisces, (2) Abell 400, (3) Abell 539, (4) Abell 1367, (5) Abell 1656 (Coma), (6) Abell 2151 (Hercules), (7) Pegasus I. From Aaronson et al. (1986) I collected the "revised" diameters, while the total corrected blue magnitudes were taken from Bothun et al. (1985), as well as the membership assignment. The mean heliocentric radial velocity of each cluster was taken from Bothun et al. (1985), then corrected and finally used to compute the absolute magnitudes M_B and diameters Δ_{25}, in kpc, via the Hubble law. Values of the Hubble constant $H_0 = 100$ km sec^{-1} Mpc^{-1} and the deceleration parameter $q_0 = 1/2$ are used throughout this paper. In the following we will refer to this sample as the *"AC"* sample (i.e., *Clusters by Aaronson et al. 1986*), followed by a number specifying a single cluster, i.e. *AC-4* will denote the cluster Abell 1367.

A very large sample of galaxies is available for our nearby cluster, Virgo: Binggeli, Sandage and Tammann (1985) provided data on the total blue magnitudes and the isophotal diameters D_{25} for many of the Virgo galaxies. To correct these data via the RC2 relations, I used their axial ratios, R_{25}, their coordinates and radial velocities. Only real members (according to the authors) were chosen. The galaxies were located at the cluster distance, estimated via the Hubble law by adopting the mean heliocentric velocity as given by Huchra (1985). I will label this data sample as *"VC"* (*Virgo Cluster*).

Two other nearby clusters, Fornax and Hydra, were chosen from "The Surface Photometry Catalogue of the ESO-Uppsala Galaxies" by Lauberts and Valentijn (1989; hereafter referred to as "ESO"). The Centaurus cluster is also present in this catalogue, yet I preferred not to include it in the my analysis, as it is a superposition of two galaxy groups (Lucey and Carter

1988). I corrected the total blue magnitudes and the D_{25} diameters as in RC2, by using the radial velocities, coordinates and axial ratios $(a/b)^0$ as listed in ESO. Only the galaxies inside the ranges in right ascension, declination, and radial velocities given in ESO were selected as cluster members. I assigned them their cluster Hubble distance, derived from the mean heliocentric velocities taken from Aaronson et al. (1981) for Fornax and from Richter, Materne and Huchtmeier (1982) for Hydra. The two data samples will be referred to as *"FC"*, and *"HC"*, respectively.

Two samples of "field" galaxies, having 21-cm line-widths, and/or distance-moduli available, were chosen in the literature. The first field sample was taken from Bottinelli, Gouguenheim, Teerikorpi (1988); from RC2 I took the blue total corrected magnitudes and isophotal diameters D^0_{25} for these galaxies, and computed their absolute magnitudes and diameters using the distance-moduli obtained by the authors via the application of the Infrared Tully-Fisher relation. These distances were rescaled to the value of the Hubble constant of 100 km sec^{-1} Mpc^{-1}, adopted in this paper. The second field sample was taken from Davis and Seaquist (1983); their blue total corrected magnitudes and D^0_{25} corrected isophotal diameters were converted to absolute magnitudes and diameters, via the blue Tully-Fisher relation, as given by Bottinelli et al. (1987). I applied the Tully-Fisher relation only to galaxies with an inclination angle larger than 30°, as suggested by the same authors. I computed the corrected line-width parameter V_{max}, suitable to the application of the adopted Tully-Fisher relation, from the 21-cm line-width parameter V_{25}, given by Davis and Seaquist (1983). For this purpose, I had to interpolate linearly between two values of the "k" parameter (see Bottinelli et al. 1983), i.e. k(V_{20}) and k(V_{40}) to obtain k(V_{25}). Once again, I had to rescale the distances by using my value of $H_0 = 100$ km sec^{-1} Mpc^{-1}. The two field data samples will be labelled *"BF"*, and *"DF"*, i.e. *Field by Bottinelli, Gouguenheim, Teerikorpi (1988)*, and *Field by Davis and Seaquist (1983)*, respectively.

The choice of these samples provided a large range of galaxy densities. In order to enlarge my set of data, I examined the *Nearby Galaxy Catalog* by Tully (1988; hereafter referred to as NBG). This catalogue gives the membership of galaxies in all degrees of the hierarchy of clustering, including clusters, groups, and looser environments (the very existence of "isolated" galaxies is in doubt; see Tully 1987). Furthermore, NBG lists the local galaxy density, "ρ", that allowed me to divide the galaxy environments into finer

ranges of galaxy density than those provided by the usual subdivisions of "fields", "groups" and "clusters". NBG collects data from several sources in the literature and applies to them the same corrections. Moreover, the catalog estimates galaxy distances, on the base of velocities and a model of velocity perturbations in the vicinity of Virgo (see, for details, Tully and Shaya, 1984). These distances are used to derive the absolute quantities M_B and Δ_{25}. All the data necessary to my analysis were taken directly from NBG; note, however, that the absolute quantities have been scaled using my choice for H_0 (instead of $H_0 = 75$ km sec^{-1} Mpc^{-1}, used in NBG). The NBG data sample will be referred as the *"T"* sample (from the name of the author, *Tully*).

A detailed list of the references to the data is given in Tables 4.1 and 4.2, for the magnitudes and the diameters, respectively: col.(1) lists the sample name and col.(2) the references. It can be seen that my data samples share, in some cases, the same references. Moreover, some galaxies are present in more than one sample; whenever there is a large enough superposition, I compared the corrected apparent magnitudes and diameters of different samples to check the consistency of my data. In particular, I found that the *"T"* samples has 89 galaxies in common with the *"VC"* sample, 84 with the *"BF"* sample, and 56 with the *"DF"*. These "double" samples were labelled: *"T-VC"*, *"T-BF"*, *"T-DF"*, when extracted from the NBG, and: *"O-VC"*, *"O-BF"*, *O-DF"*, when extracted from the other data samples, with *"T"* denoting *"Tully"*, and *"O"*, *"Others"*.

In Fig.4.3a,c,e I plotted the differences, ΔB_T^0, between the two values of the corrected apparent magnitudes, for each galaxy in common to the *"T"* and *"O"* samples, vs. the average, $< B_T^0 >$, of the two values; similar plots are shown in Fig.4.3b,d,f for the logarithm of the corrected apparent diameter $\log D_{25}^0$. The statistical analysis confirmed the visual impression that no significant correlation is present, neither between the values of ΔB_T^0 and $< B_T^0 >$, nor between the values of $\Delta log D_{25}^0$ and $< \log D_{25}^0 >$, so there is no strong evidence for bias. The shift in the y-axes, evident from the figures, is due to the different corrections applied to the data (a major source of difference being the adopted values for the absorption of our galaxy in NBG and RC2). The different corrections applied to the data, and the possible differences in the distances used to derive absolute quantities, suggested me to consider the *"T"* samples separately from the others.

In order to fulfil the comparative analysis of the L-D relations of these

Table 4.1: Magnitude Errors

Sample (1)	Data Reference (2)	σ_{B_T} (3)	Error Reference (4)
AC	Bothun et al. (1985)	0.2	Bothun et al. (1985)
VC	1: de Vaucouleurs and Pence (1979)	0.14	Binggeli, Sandage, Tammann (1985)
VC	2: de Vaucouleurs and Pence (1979)	0.34	Binggeli, Sandage, Tammann (1985)
VC	3: Binggeli, Sandage, Tarenghi (1984)	0.10	Binggeli, Sandage, Tammann (1985)
VC	4: Binggeli, Sandage, Tarenghi (1984)	0.10	Binggeli, Sandage, Tammann (1985)
VC	5: Karachentsev and Karachentseva (1982)	0.20	Binggeli, Sandage, Tammann (1985)
VC	6: Average from sources 1 and 4	0.26	Average error from sources 1 and 4
VC	7: Average from sources 2 and 4	0.30	Average error from sources 2 and 4
VC	8: Average from sources 5 and 4	0.27	Average error from sources 5 and 4
FC and HC	ESO	0.12	ESO
BF	RC2	0.09	RC2
DF	V: RC2	0.09	RC2
DF	H: Harvard magnitudes from RC2	0.35	RC2
DF	Z: Zwicky et al. (1961–1968)	0.39	de Vaucouleurs and Pence (1979)
T	1: Holmberg (1958)	0.11	RC2
T	2: RC2	0.09	RC2
T	3: Zwicky et al. (1961–1968)	0.39	de Vaucouleurs and Pence (1979)
T	5: Harvard magnitudes from RC2	0.35	RC2
T	7: de Vaucouleurs, de Vaucouleurs, Buta (1981)	0.114	de Vaucouleurs, de Vaucouleurs, Buta (1981)

Col.(1): Sample identification label;

Col.(2): references to the magnitude data;

Col.(3): error on the magnitude, σ_{B_T};

Col.(4): references to the assumed error.

Table 4.2: Diameter Errors

Sample (1)	Data Reference (2)	σ (3)	Error Reference (4)
AC	GASP: Aaronson et al. (1986)	$2''.4$	Cornell et al. (1987)
AC	UGC: Aaronson et al. (1986)	0.071	Cornell et al. (1987)
VC	D_{25}: Binggeli, Sandage, Tammann (1985)	0.04	RC2
FC and HC	ESO	0.04	ESO
BF	RC2	0.04	RC2
DF	V: RC2	0.04	RC2
DF	U: Nilsson (1973)	0.05	RC2
T	2: Nilsson (1973)	0.05	RC2
T	4: Vorontsov-Velyaminov, Krasnogorskaya, Arkipova (1962–74)	0.06	RC2
T	5: de Vaucouleurs, de Vaucouleurs (1964)	0.05	de Vaucouleurs, de Vaucouleurs (1964)
T	6: Lauberts (1982)	0.04	ESO
T	9: Fouqué, Paturel (1983)	0.027	Fouqué, Paturel (1983)

Col.(1): Sample identification label;

Col.(2): references to the magnitude data;

Col.(3): error on the logarithm of the diameter, $\sigma_{\log D_{25}}$, or error in arcsec on the diameter, $\sigma_{D_{25}}$, when the value has the superscript $''$;

Col.(4): references to the assumed error.

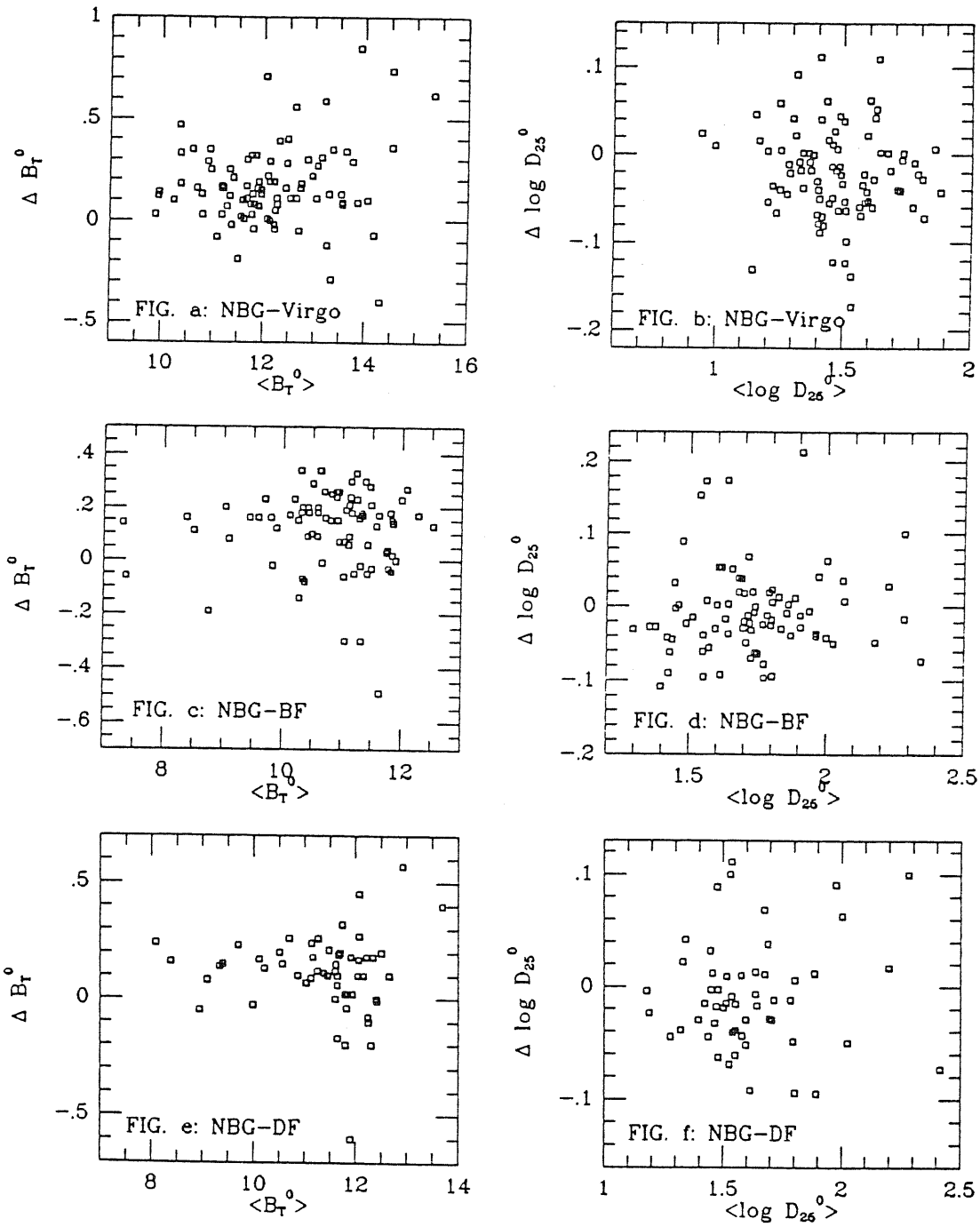

Figure 4.3: a-f) Comparison of corrected apparent magnitudes and diameters for galaxies in different samples. Fig.4.3a,c,e): differences between the values of the corrected apparent magnitudes B_T^0, for each galaxy in common to the compared samples, vs. the average of the two values. Fig.4.3b,d,f): similar plots for the logarithm of the corrected apparent diameter $\log D_{25}^0$, in place of the magnitude.

samples, I needed to know the uncertainties to be assigned to both the quantities involved in the relation. The uncertainties in the data vary according to their source; I list them in Tables 4.1 and 4.2, for the magnitudes and the diameters respectively: col.(1) lists the sample name; col.(2) lists the references to the data; col.(3) contains the values of the uncertainties on magnitudes (Table 4.1) and on $\log D_{25}$ (Table 4.2), except when the value has the superscript ", in which case the error is in seconds of arc on D_{25}; col.(4) lists the reference to the paper from which I took the estimates of the uncertainties. The uncertainties listed do not include the errors in the distance-estimates; these are negligible compared with the data-uncertainties, when one is dealing with galaxies in the same cluster. As far as the field galaxies are concerned, I assumed distance errors = 20%, typical of good distance-estimates from the Tully-Fisher relation (see, e.g., Bottinelli et al. 1983, Bertschinger et al. 1990, Biviano et al. 1990). No error was assigned to the distance-estimates in NBG, because the various subsamples I compared would be equally affected by these errors, and I am not interested in the absolute values of the parameters of the L-D relation; moreover, an estimate of this error is not a trivial task. Table 4.3 , lists the mean distances of the clusters considered in our analysis: col.(1) lists the cluster name, col.(2) its mean radial heliocentric velocity, $V_{\odot}$, col.(3) the distance-modulus derived via the Hubble relation, using the velocity given in col.(2) after the application of the corrections for the local motions. See Tully (1987) for the NBG clusters.

4.3 The Statistical Methodology

I investigated the L-D relation in its logarithmic form, M_B vs. $\log \Delta_{25}$.

I suppose that the intrinsic relation of M_B vs. $\log \Delta_{25}$ is linear, so I thought to fit the data with a straight regression line. The method most widely used is the standard least-squares fit: let y_i, x_i be the observations; the fitted line is

$$u(x) = q + p \cdot x \tag{4.1}$$

with the slope and the intercept given by

$$p = [\Sigma w_i \Sigma w_i x_i y_i - \Sigma w_i x_i \Sigma w_i y_i]/D \tag{4.2}$$

Table 4.3: Cluster Parameters

Cluster Name (1)	$< V_{\odot} >$ (2)	Distance Modulus (3)
Virgo	1150	30.54
Pisces	5271	33.64
Abell 400	7240	34.28
Abell 539	8535	34.64
Abell 1367	6426	34.09
Coma	6950	34.27
Abell 2151	10998	35.24
Pegasus	4168	33.10
Abell 2634/66	8936	34.78
Fornax	1340	30.17
Hydra	3707	32.78

Col.(1): Cluster Name;

Col.(2): Mean radial heliocentric velocity of the cluster;

Col.(3): Cluster distance modulus.

$$q = [-\Sigma w_i x_i \Sigma w_i x_i y_i + \Sigma w_i x_i^2 \Sigma w_i y_i]/D \tag{4.3}$$

where $D = [\Sigma w_i \Sigma w_i x_i^2 - (\Sigma w_i x_i)^2]$ and w_i is the weight of the i-th observation.

But a more exact estimate can only be made if one of the three quantities σ_γ (standard error of x), σ_δ (standard error of y), or $k = (\sigma_\delta/\sigma_\gamma)^2$, is known. According to the theory of regression (see e.g. Guest, 1961), let the observed x and y ($\log \Delta_{25}$ and M_B) differ from the true values x' and y'. The errors are $\gamma = x - x'$ and $\delta = y - y'$. The basic assumption is that the errors γ and δ are random variables. If one supposes that the error-free points x_i' and y_i' lie exactly on the straight line $y = Q + P \cdot x$, the estimates of P are:

i) when I consider only the error on x:

$$p_{(y)} = \Sigma w_i x_i y_i / (\Sigma w_i x_i^2 - \Sigma w_i' \sigma_{\gamma i}^2) \tag{4.4}$$

ii) when I consider only the error on y:

$$p_{(x)} = (\Sigma w_i y_i^2 - \Sigma w_i' \sigma_{\delta i}^2)/\Sigma w_i x_i y_i \tag{4.5}$$

iii) when I considered the error on both the variables x and y:

$$p = m + \sqrt{(m^2 + k)} \tag{4.6}$$

where

$$m = (\Sigma y_i^2 - k \Sigma w_i x_i^2)/2\Sigma w_i x_i y_i \tag{4.7}$$

$$w_i = \sigma^2/\sigma_{\gamma i}^2 (k_i + P^2) \tag{4.8}$$

with $k_i = \sigma_{\delta i}^2/\sigma_{\gamma i}^2$

$$w_i' = w_i(1 - w_i/\Sigma w_i) \tag{4.9}$$

$$k = \Sigma w_i' \sigma_{\delta i}^2 / \Sigma w_i' \sigma_{\gamma i}^2 \tag{4.10}$$

The estimated standard deviation is

$$\sigma^2 = w_i \sigma_{\gamma i}^2 (k_i + P^2) \tag{4.11}$$

Similar formulae exist for the intercept.

In their recent paper, Giuricin et al. (1989) adopted the regression line with only the errors on x (that is the $\log \Delta_{25}$) that are the larger ones. Another possibility (see e.g. Peterson, Srom and Strom 1979) is the use of standard regression lines without errors, but I think that this procedure underestimates the errors associated with the parameters of the fitted lines. In conclusion, to each data sample I preferred to fit the straight regression line, obtained via the least squares method applied to data affected by errors on both axes (eq.4.6 for the slope) (see e.g. Guest 1961; for a recent astrophysical application, see, e.g., Biviano et al. 1990). In this way I weighted appropriately each datum, according to its internal accuracy.

The comparison of different regression lines may be given a statistical meaning via the use of two tests: the *Welch* test, that applies when only two lines are involved, and the *Homogeneity* (or *Variance-ratio*) test, which applies in all other cases (see, e.g., Guest 1961; for recent astrophysical applications, see, e.g., Giuricin et al. 1989, Biviano et al. 1990).

The Welch test is used when two different determinations p' and $p"$ of the slope P (similarly two different determination of intercept) are made. The ratio

$$t = \frac{p' - p"}{[\sigma^2(p') + \sigma^2(p")]^{1/2}} \tag{4.12}$$

will be distributed as a student t with a number of degrees of freedom given by $\nu' + \nu" = n' + n" - 4$; σ is the estimated standard deviation of the slope. Similarly, for the two estimates of the intercepts the ratio

$$\frac{q' - q"}{[\sigma^2(q') + \sigma^2(q")]^{1/2}} \tag{4.13}$$

is used; now σ is the estimated standard deviation of the intercept.

If more then two different determinations of the straight line are made, the homogeneity test becomes useful. Let r different sets of observations y_{ji} are made, a straight line may be fitted to each separately. The distribution of the slopes p will now be investigated, on the assumption that σ^2 and the true slope P is the same in each set. The weighted mean of all the slopes will be

$$< p >= \Sigma W_j p_j / \Sigma W_j \tag{4.14}$$

where $W_j \propto 1/\sigma(p_j)$, and $\sigma(p_j)$ is the the estimated deviation of the j-th slope. On the postulate of homogeneity of slopes and standard deviations σ the ratio

$$F = \frac{\frac{\Sigma W_j (p_j - <p>)^2}{r-1}}{\frac{\Sigma\Sigma w_{ji} v_{ji}^2}{n-2r}} \tag{4.15}$$

will be distribuited as F with $(r-1, n-2r)$ degrees of freedom. This provides a test for the homogeneity of the slopes. If the values of the slopes pass the test for homogeneity, $< p >$ will provide an estimate of P, with an estimated standard deviation given by the equation

$$\sigma^2(< p >) = \sigma^2/\Sigma W_j \tag{4.16}$$

The homogeneity of the values q can be tested in a similar way. If the values q and p both pass the homogeneity test, the lines may be assumed to be all estimates of the same straight line.

Other authors (see e.g. Giuricin et al. 1985, 1988) applied the Welch test even when more than two samples are compared, but they compared the lines one to one. However, I thought that the homogeneity test was more suitable to comparisons between more than two samples.

So, in conclusion, I used Welch test when I considered only two samples and I used the homogeneity test when I considered more than two samples. These tests yielded the probabilities "P(q)" and "P(p)" that the differences in the intercepts "q" and slopes "p" of the compared fitting lines are significant. Summarizing, the results of the line-fitting have been collected in Table 4.4: col.(1) lists the sample identification label; col.(2) lists the number of galaxies considered in the sample; col.(3) lists the value of the intercept "q" of the regression line fitted to the data, M_B vs. $\log \Delta_{25}$, followed by its associated error "σ_q" in parentheses; col.(4) lists the value of the slope "p" of the same regression line, followed by its associated error "σ_p", in parentheses; col.(5) gives a few words of comment to allow an easier identification of the sample considered. The results of the comparison analyses have been listed in Table 4.5: col.(1) lists the progressive number of the result obtained, col.(2) lists the samples that have been compared, cols.(3) and (4) list the values of P(q) and P(p) in % obtained via the use of the homogeneity test or the Welch test; col.(5) gives a few words of comment to allow an easy identification of the samples involved in the analysis performed.

Table 4.4 Results of the line-fitting

Sample (1)	# of Gal. (2)	q (σ_q) (3)	p (σ_p) (4)	Notes (5)
AC-1	18	-11.9 (1.6)	-5.9 (1.3)	Pisces cluster
AC-2	7	-15.3 (1.6)	-3.4 (1.2)	Abell 400 cluster
AC-3	5	-17.3 (2.8)	-2.7 (2.2)	Abell 539 cluster
AC-4	17	-14.3 (0.8)	-4.2 (0.7)	Abell 1367 cluster
AC-5	12	-13.7 (4.2)	-4.6 (3.1)	Coma cluster
AC-6	8	-15.7 (1.9)	-3.2 (1.4)	Abell 2151 cluster
AC-7	19	-12.8 (1.2)	-5.0 (1.1)	Pegasus I cluster
VC	177	-12.5 (0.1)	-5.7 (0.2)	Virgo cluster
VC-I	48	-12.5 (0.2)	-5.8 (0.2)	Virgo, inner shell
VC-M	29	-12.5 (0.2)	-5.8 (0.2)	Virgo, middle shell
VC-O	29	-12.5 (0.3)	-5.7 (0.4)	Virgo, outer shell
HC	83	-13.8 (0.3)	-4.7 (0.2)	Hydra cluster
HC-I	25	-13.2 (0.4)	-5.2 (0.4)	Hydra, inner shell
HC-M	29	-14.2 (0.4)	-4.4 (0.4)	Hydra, middle shell
HC-O	29	-14.1 (0.5)	-4.4 (0.4)	Hydra, outer shell
FC	58	-12.9 (0.2)	-5.5 (0.2)	Fornax cluster
FC-I	31	-12.7 (0.2)	-5.8 (0.3)	Fornax, inner shell
FC-MO	27	-13.1 (0.2)	-5.3 (0.3)	Fornax, middle/outer shell
BF	86	-13.1 (0.3)	-5.4 (0.3)	Bottinelli, Gouguenheim, Teerikorpi (1988) Field
DF	70	-13.7 (0.4)	-4.7 (0.3)	Davis, Seaquist (1983) Field
DF-N	56	-13.2 (0.5)	-5.1 (0.4)	Davis, Seaquist (1983) Field, Nearby sample

Table 4.4 Continued

Sample (1)	# of Gal. (2)	q (σ_q) (3)	p (σ_p) (4)	Notes (5)
TC-1	100	-12.9 (0.3)	-5.5 (0.3)	NBG cluster Virgo
TC-2	54	-12.0 (0.7)	-6.1 (0.7)	NBG cluster Ursa Major
TC-3	19	-13.9 (0.3)	-4.3 (0.3)	NBG cluster Coma I
TC-4	13	-12.3 (1.2)	-5.7 (0.9)	NBG cluster NGC 5371
TF	520	-12.5 (0.1)	-5.6 (0.1)	NBG Field galaxies
TG	758	-12.2 (0.1)	-5.8 (0.1)	NBG Group galaxies
TC	238	-13.0 (0.2)	-5.4 (0.2)	NBG Cluster galaxies
Tρ-8	396	-12.4 (0.2)	-5.7 (0.2)	NBG, $\log\rho \leq -8$
Tρ-6	354	-12.6 (0.2)	-5.6 (0.2)	NBG, $-8 < \log\rho \leq -6$
Tρ-4	278	-12.1 (0.2)	-5.9 (0.2)	NBG, $-6 < \log\rho \leq -4$
Tρ-2	193	-12.4 (0.2)	-5.7 (0.2)	NBG, $-4 < \log\rho \leq -2$
Tρ0	122	-13.0 (0.3)	-5.3 (0.3)	NBG, $-2 < \log\rho \leq 0$
Tρ+	173	-12.6 (0.3)	-5.7 (0.3)	NBG, $\log\rho > 0$
O-VC	89	-12.5 (0.3)	-5.7 (0.3)	Virgo double
O-BF	84	-13.4 (0.4)	-5.0 (0.4)	Davis, Seaquist (1983) Field double
O-DF	56	-13.1 (0.3)	-5.4 (0.3)	Bottinelli, Gouguenheim, Teerikorpi (1983) Field double
T-VC	89	-12.7 (0.4)	-5.6 (0.4)	NBG Virgo double
T-BF	84	-13.2 (0.3)	-5.2 (0.3)	NBG Davis, Seaquist (1983) Field double
T-DF	56	-12.3 (0.5)	-5.9 (0.4)	NBG Bottinelli, Gouguenheim, Teerikorpi (1988) Field double

Col.(1): Sample identification label;

Col.(2): number of galaxies in the sample;

Col.(3): intercept of the fitted line, q, and its associated error, σ_q, in parentheses;

Col.(4): slope of the fitted line, p, and its associated error, σ_p, in parentheses;

Col.(5): notes on the samples considered.

4.4 The Results of Analysis

4.4.1 The Cluster Environments

I started by comparing Aaronson's clusters. They span the whole range of galaxy densities typical of a cluster environment: from a loose cluster, like Pisces, to a rich one, like Coma. It is evident from result no.1 in Table 4.5 that the homogeneity is quite large. Nevertheless, the lines fitted to the data samples with very few galaxies have a low statistical significance (see Table 4.4), so, hereafter, in my analysis I will consider only samples containing at least 10 galaxies. Table 4.5 shows that the homogeneity still holds for the four richest samples only (see result no.2). In Fig.4.4(a-d)I plotted these data samples and their fitting lines.

I next considered the Virgo cluster sample. Since the *VC* data sample is large enough, I subdivided it into three subsamples of galaxies located at different projected distances from the cluster center. In this way, I selected regions of different average galaxy densities. I labelled as *"VC-I", "VC-M", "VC-O",* the subsamples of galaxies in the *Inner, Middle, and Outer* shells, respectively, defined by these limits:

(1) Inner shell, dist $\leq$ 0.5 Mpc;

(2) Middle shell, 0.5 $<$ dist $\leq$ 1 Mpc;

(3) Outer shell, dist $>$ 1 Mpc;

where "dist" is the distance from the cluster center, projected onto the plane of the sky. The three subsamples showed extremely similar L-D relations (see Table 4.4 and result no.3 of Table 4.5). The subsamples are shown in Fig.4.5, plotted with different symbols; the line fitting the whole data sample is also plotted.

The same partition was made for the samples *HC* and *FC*. Note, however, that since no more than four galaxies were present in the *FC-O* subsample, I linked it to *FC-M*: the combined sample was named *FC-MO*. Again, no significant differences were present (see Table 4.5, results no.4 and 5; the subsamples are plotted in Fig.4.6a,b).

Next I compared the well sampled clusters all together: *AC-1, AC-4, AC-5, AC-7, VC, HC, FC*. The homogeneity was not found to be as large as for the *AC* samples alone (compare, in Table 4.5, results no.2 and no.6); this was

Table 4.5 Results of the Comparative Analyses

Result no. (1)	Samples considered (2)	P(q) (3)	P(p) (4)	Notes (5)
1	AC-1, AC-2, AC-3, AC-4, AC-5, AC-6, AC-7	3.0	1.2	Aaronson et al. (1986) clusters
2	AC-1, AC-4, AC-5, AC-7	7.1	5.2	Well sampled Aaronson et al. (1986) clusters
3	VC-I, VC-M, VC-O	0.3	0.6	Virgo cluster galaxies in different shells
4	HC-I, HC-M, HC-O	51.4	51.3	Hydra cluster galaxies in different shells
5	FC-I, FC-MO	88.2	89.4	Fornax cluster galaxies in different shells
6	AC-1, AC-4, AC-5, AC-7, VC, HC, FC	73.0	60.8	All well sampled clusters
7	BF, DF	88.6	95.8	Field samples
8	BF, DF-N	57.1	70.8	Bottinelli, Gouguenheim, Teerikorpi (1988) field and nearby galaxies in Davis, Seaquist (1983) field
9	AC-1, AC-4, AC-5, AC-7, VC, HC, FC, BF, DF	84.9	75.9	All clusters and field samples

Table 4.5 Continued

Result no. (1)	Samples considered (2)	P(q) (3)	P(p) (4)	Notes (5)
10	TC-1, TC-2, TC-3, TC-4	58.1	58.6	NBG clusters
11	TF, TG, TC	50.7	32.5	NBG galaxies in the field, groups and clusters
12	Tρ-8, Tρ-6, Tρ-4, Tρ-2, Tρ0, Tρ+	5.7	1.2	NBG galaxies at different local densities
13	O-VC, O-BF, O-DF	61.2	54.1	double samples
14	T-VC, T-BF, T-DF	25.1	19.4	NBG double samples

Col.(1): Progressive number;

Col.(2): samples compared;

Col.(3): probability P(q) in % that the values of the intercepts of the compared fitting lines are not all estimates of the same intercept;

Col.(4): probability P(p) in % that the values of the slopes of the compared fitting lines are not all estimates of the same slope;

Col.(5): notes on the samples considered.

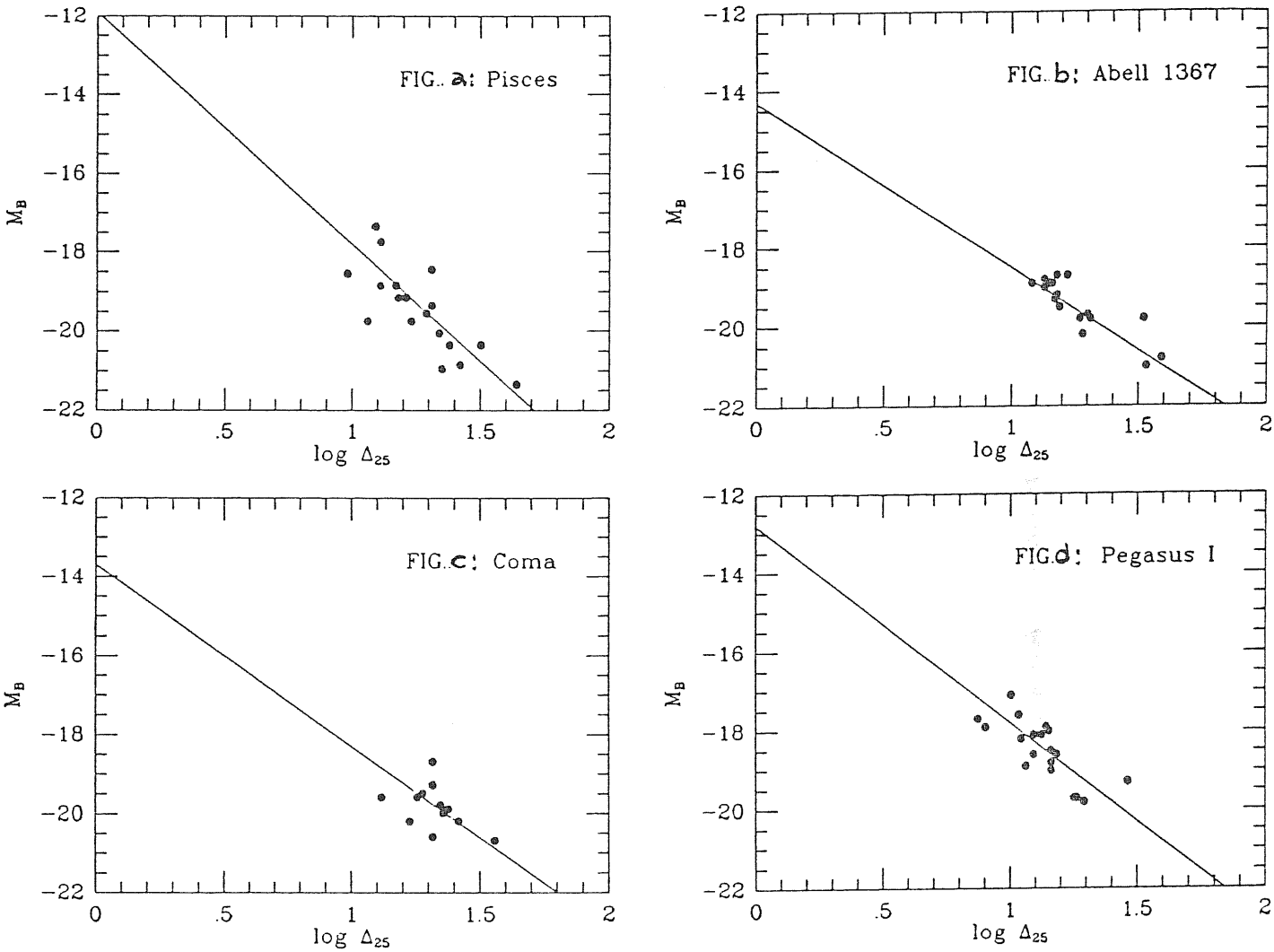

Figure 4.4: a-d) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): clusters by Aaronson et al. (1986); the solid lines are the lines fitting the data.

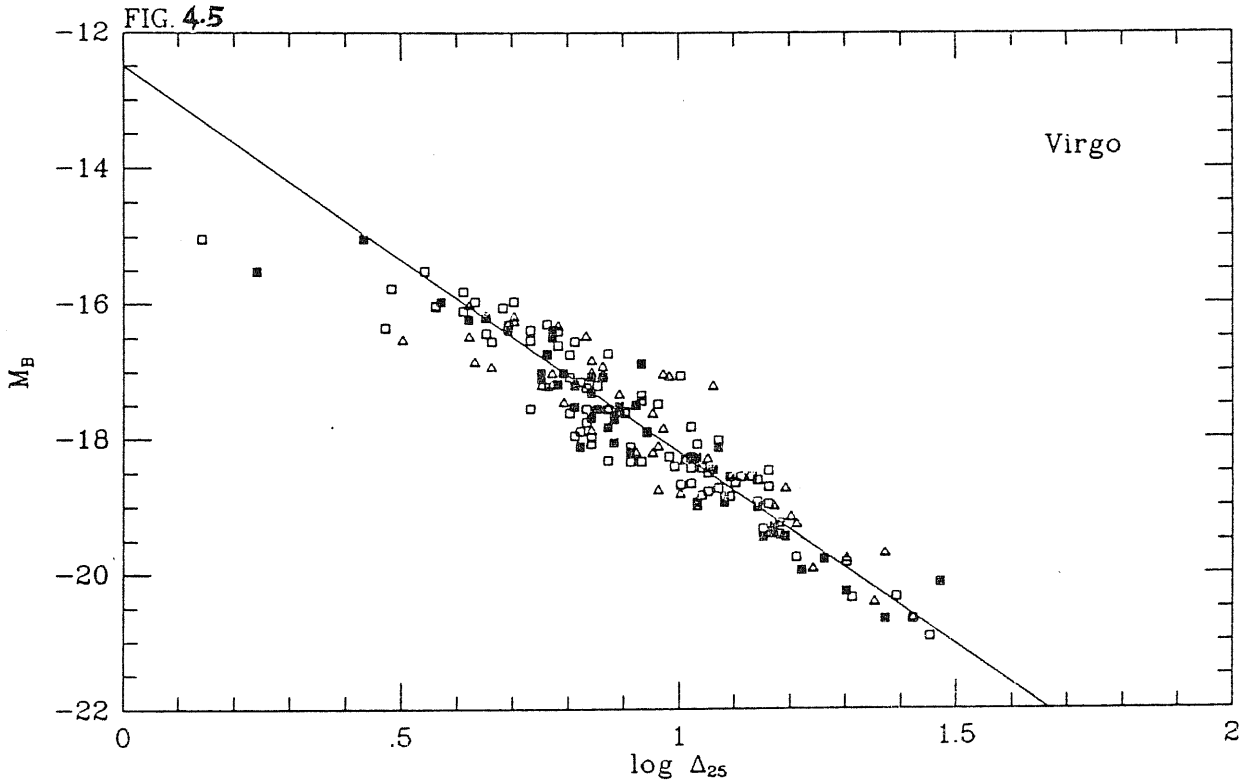

Figure 4.5: Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): Virgo cluster; filled squares, open squares, open triangles represent galaxies in the inner, middle, and outer shells, respectively; the solid line is the line fitting the whole sample.

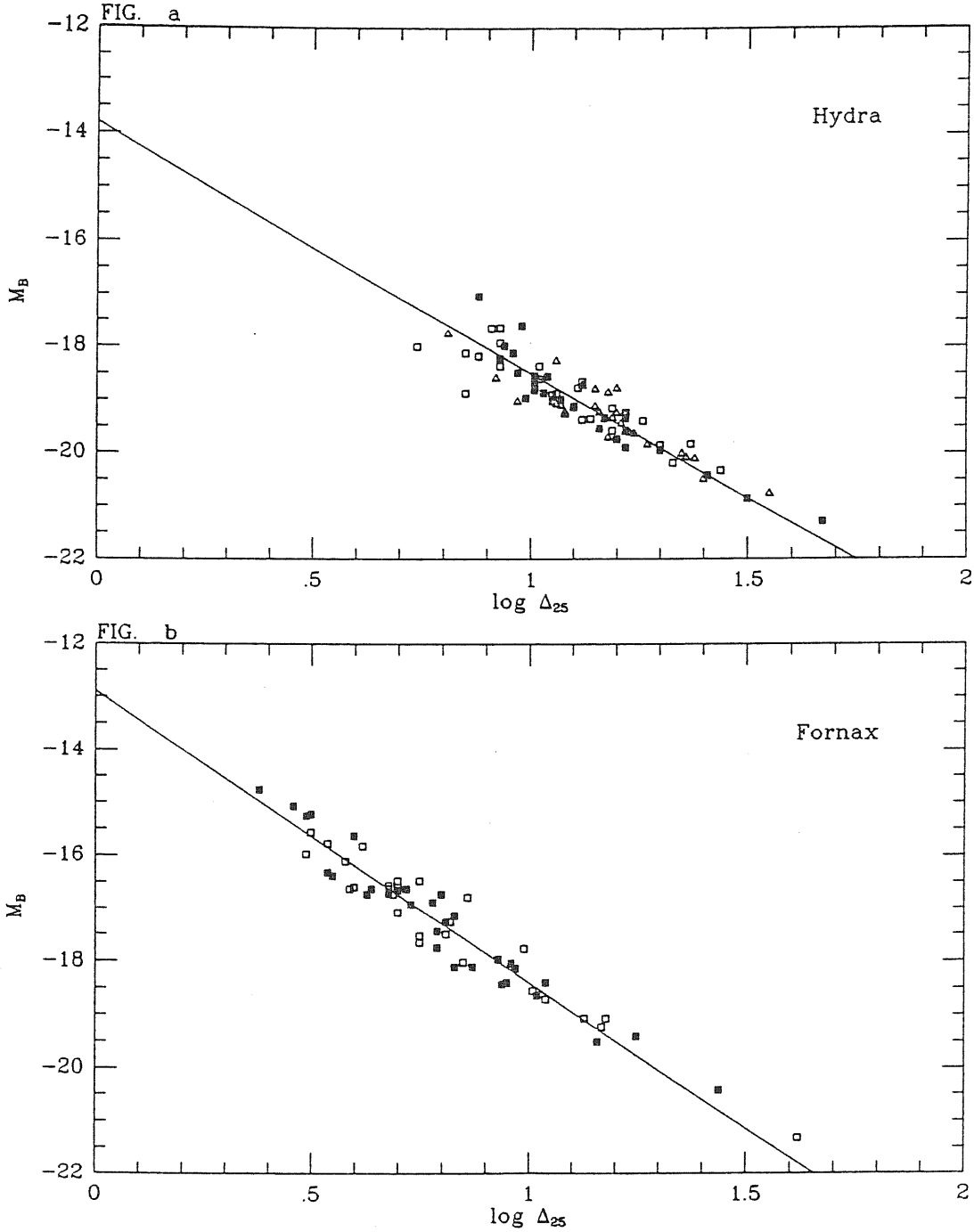

Figure 4.6: a) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): Hydra cluster; filled squares, open squares, open triangles represent galaxies in the inner, middle, and outer shells, respectively; the solid line is the line fitting the whole sample. b) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): Fornax cluster; filled squares and open squares represent galaxies in the inner and middle-outer shells, respectively; the solid line is the line fitting the whole samples.

to be expected, when comparing samples taken from different authors, who selected and corrected data in different ways. Nevertheless, the differences were not significant.

4.4.2 The Field Environments

The comparison of the two field samples showed a partially significant difference in the slopes (at the 96 % confidence level; see Table 4.5, result no.7). I noted that the *DF* sample had more galaxies of very large luminosities and diameters than the *BF* sample (see Fig.4.7a-b, plotting the two data samples and their fitting lines). The difference is amenable to a selection bias in the sample compilation; in fact, the *DF* sample is partially made up of data from the "Uppsala General Catalogue of Galaxies" by Nilson (1973; hereafter referred to as "UGC"). UGC is a diameter-limited catalogue; hence, it selects larger and larger galaxies with increasing distance. This bias is clearly present in the *DF* sample, coupled to the Malmquist bias on the absolute magnitudes: at large distances, and thus at the bright end of the L-D relation, the sample lacks small diameter galaxies. This bias forced a shallower fitting line. I tried to overcome this problem by discarding the most distant galaxies of the *DF* sample, i.e. galaxies with $V_{\odot} > 2400$ km sec^{-1}, the same upper limit on radial velocity as in the *BF* sample. This *DF* restricted sample was labelled *"DF-N"*, (the suffix *"-N"* indicates the selection of *"Nearby"* galaxies only in the sample). The result of the comparison between *BF* and *DF-N* can be seen in the relevant tables (see result no.8): no significant difference was left (see also Fig.4.7b: the dashed line in Fig.4.7b is the line fitting the data sample *DF-N*).

The L-D relations of the field samples were found to be homogeneous also with those of the clusters: the homogeneity test yielded no significant values of P(q) and P(p) ($< 90\%$, see result no.9 in Table 4.5).

4.4.3 The NBG Sample

I considered the *T* sample separately, since it is a homogeneous sample (as explained in the previous section). I started by considering the clusters identified by Tully (1987) in NBG. Only 10 samples had sufficient data to allow me to perform the analysis, and, of these, four had more than 10 galaxies, i.e. clusters: (1) Virgo, (2) Ursa Major, (3) Coma I, (4) NGC 5371. The

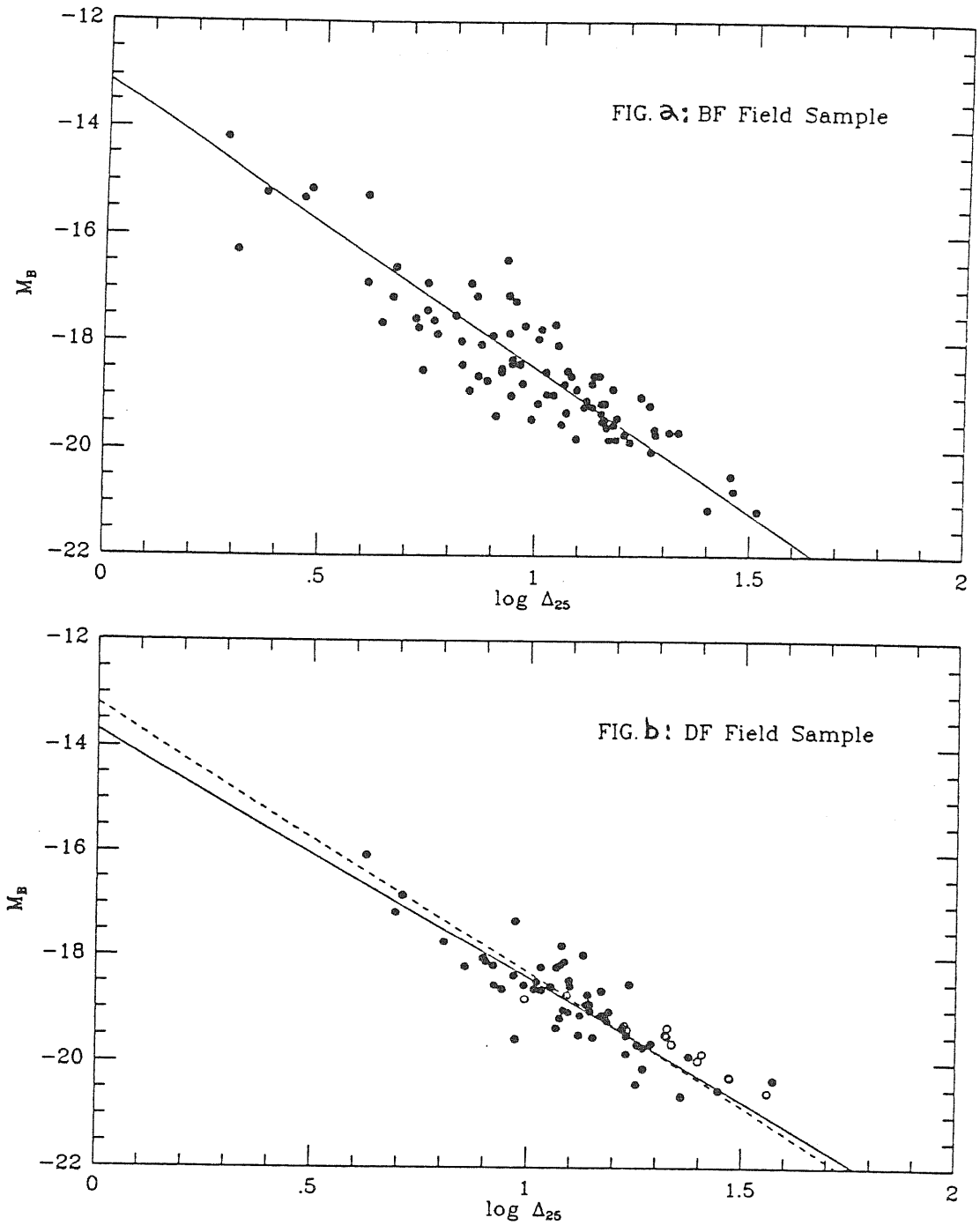

Figure 4.7: a) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc):: field sample of Bottinelli, Gouguenheim, Teerikorpi (1988); the solid line is the line fitting the data. b) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): field sample of Davis and Seaquist (1983); filled and open circles represent galaxies with radial velocities, respectively, lower and larger than 2400 km/sec. The solid line is the line fitting the whole sample, the dashed line is the line fitting the sample of nearby galaxies only.

cluster samples were found to be homogeneous in their L-D relations (see result no.10 in Table 4.5; the cluster samples have been labelled *"TC-j"*, with $j = 1, \ldots, 4$). Fig.4.8(a-d) show the data samples and the corresponding fitting lines. The inclusion of the other 6 clusters with few data did not affect this result.

Since all the cluster samples were found to be homogeneous, I felt confident to combine their data in a single sample of cluster galaxies, labelled *"TC"*. I compared it with a sample collecting all the galaxies located in groups, labelled *"TG"*, and another sample of "field" galaxies, formed of all galaxies which are neither cluster nor group members; I labelled this sample *"TF"*. These samples have significantly different mean densities $< \rho >$ being:

$$< \rho >= \begin{cases} 1.74 \pm 0.07 & \ldots TC \text{ sample} \\ 0.32 \pm 0.01 & \ldots TG \text{ sample} \\ 0.21 \pm 0.01 & \ldots TF \text{ sample} \end{cases} \tag{4.17}$$

in units of galaxies/Mpc^3 (see NBG). The values of the line parameters are listed in Table 4.5.4 and the result of the homogeneity test in Table 4.5: the three subsamples are not significantly different from one another. The three data samples are plotted in Fig.4.9a-c, with the respective fitting lines.

Since NBG lists the local density of every galaxy, I examined a finer subdivision into ρ than the one considered previously. I divided the whole sample into intervals of values of $\log \rho$, chosen in order to keep a (similar) large enough amount of data in each subsample. I have labelled these subsamples as follows:

(1) *"Tρ-8"*: galaxies with $\log \rho \leq -8$;

(2) *"Tρ-6"*: galaxies with $-8 < \log \rho \leq -6$;

(3) *"Tρ-4"*: galaxies with $-6 < \log \rho \leq -4$;

(4) *"Tρ-2"*: galaxies with $-4 < \log \rho \leq -2$;

(5) *"Tρ0"*: galaxies with $-2 < \log \rho \leq 0$;

(6) *"Tρ+"*: galaxies with $\log \rho > 0$.

The comparison of these subsamples showed a very high degree of homogeneity (see Table 4.4 for the values of the fitting line parameters, and Table

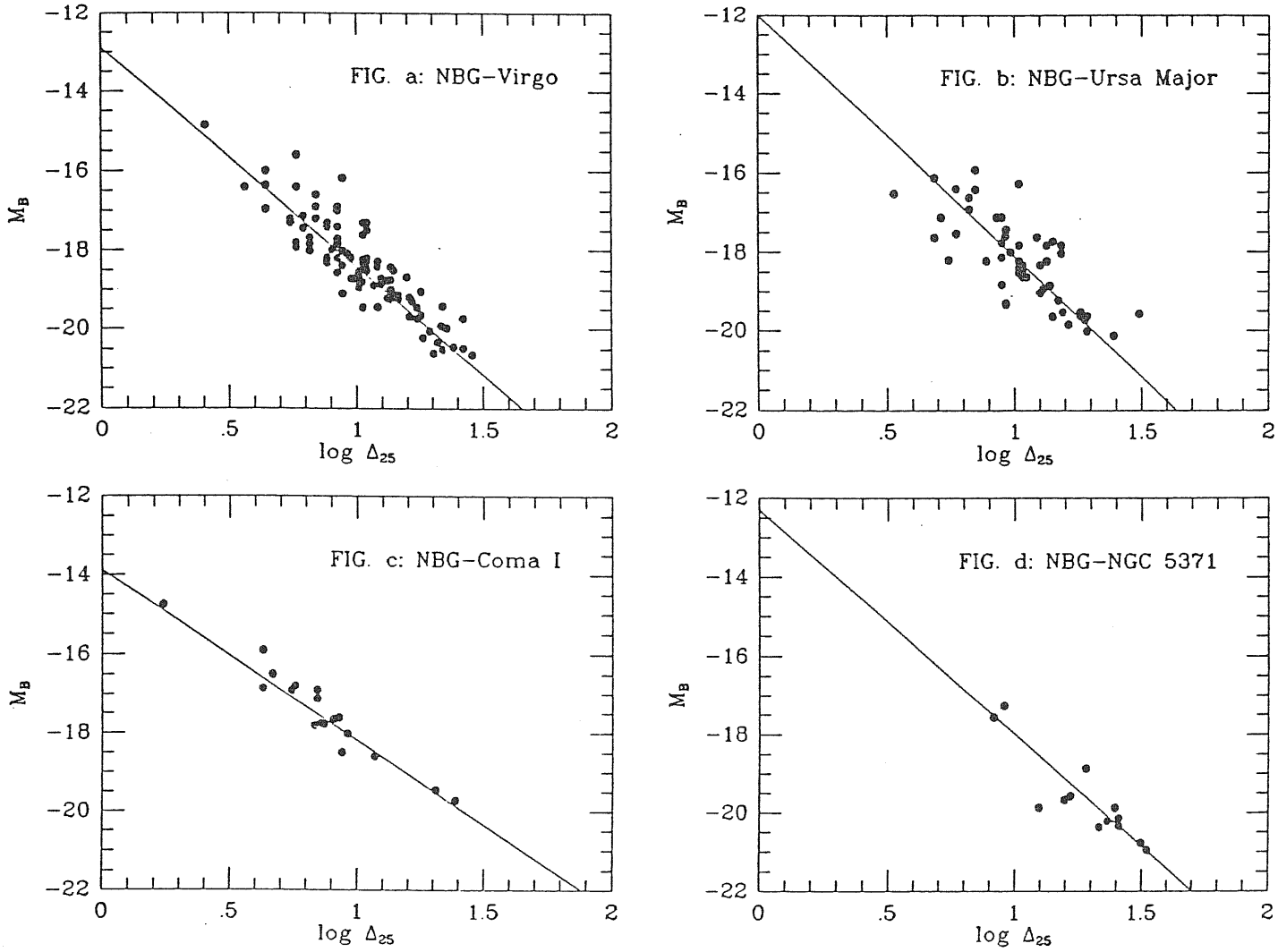

Figure 4.8: a-d) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): NBG clusters; the solid lines are the lines fitting the data.

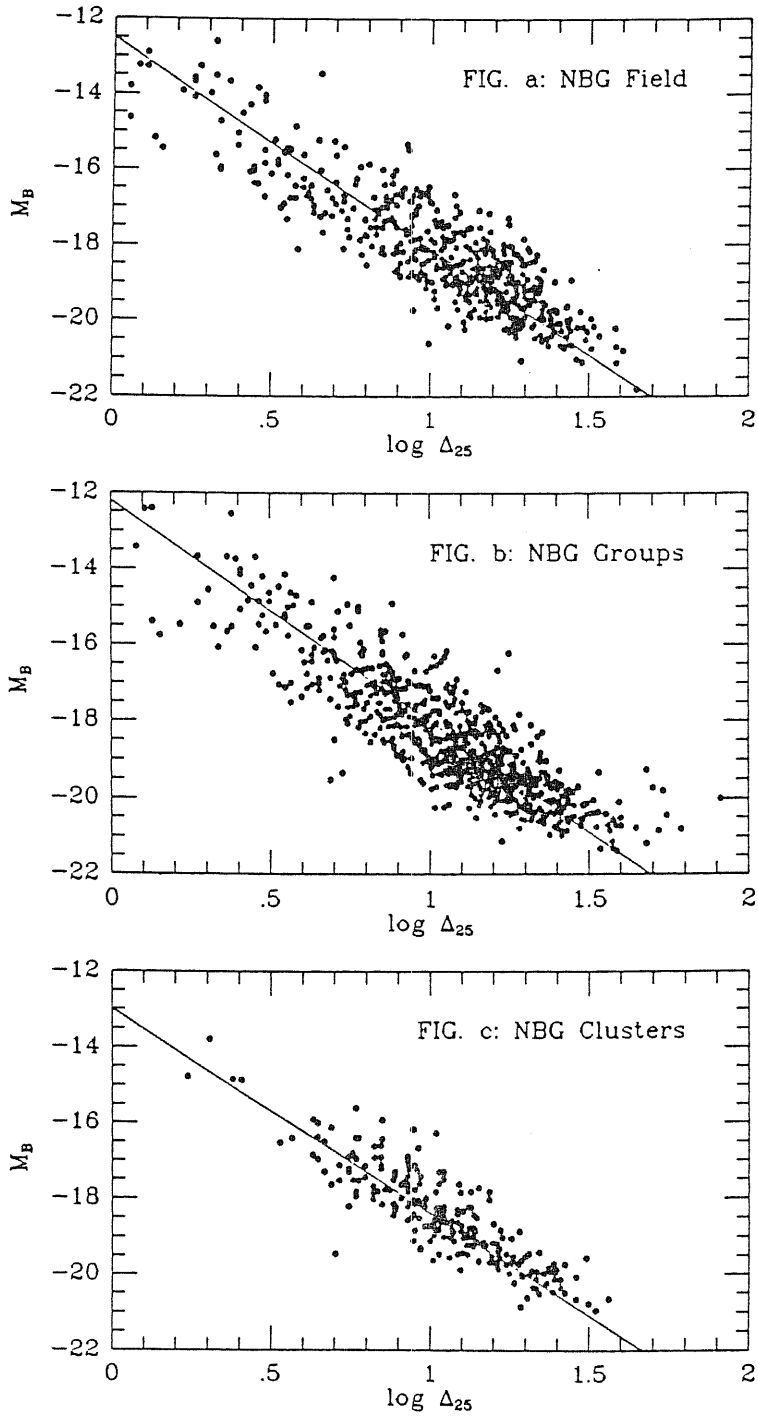

Figure 4.9: a-c) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): NBG galaxy samples in the field, groups, and clusters; the solid lines are the lines fitting the data.

4.5 for the results of the homogeneity test): the probability that these lines are not sample estimates of the same "true" line, is less than 10 %. The similarity of the L-D relations can be appreciated by looking at Fig.4.10a-f, where these data samples are plotted.

4.4.4 Inhomogeneity Effects

Part of the differences observed in the L-D relations of different samples could be ascribed to differences in the criteria used for selection and reduction of the data. In order to verify this, I compared the "double" samples *"O-VC"*, *"O-BF"*, *"O-DF"*, with each other and, separately, the "double" samples *"T-VC"*, *"T-BF"*, *"T-DF"*, with each other. These samples contain the same galaxies (see section 4.2), yet the data in the *"T"* samples are taken from the same catalog (NBG), while this is not true for the data in the *"O"* samples. Hence, the absolute corrected magnitudes and diameters of the *"T"* samples, have been obtained from the observed quantities in a more homogeneous way.

The comparisons showed that, although the values of the line parameters did not change very much from the *"O"* to the *"T"* samples (see Table 4.4), the homogeneity was larger in the *"T"* samples (compare, in Table 4.5, result no.13 and no.14). Thus, it is possible that part of the differences in the L-D relations arose because of the inhomogeneity of the data samples.

4.5 Discussion

I analyzed the L-D relation for samples of galaxies located in environments of different densities. The differences among the different relations were not found to be statistically significant. This result applies both to the cluster environments alone and to the field, group and cluster environment taken together. In the same cluster different regions can have different mean densities, so I also compared the L-D relation for galaxies located at different distance from their cluster center; again, a large homogeneity was found. When I used the local density parameter to discriminate among different environments, the L-D relations showed no differences either. These results showed that any possible environmental effect is not strong enough to affect significantly the L-D relation for disk galaxies, in the samples used in the present paper.

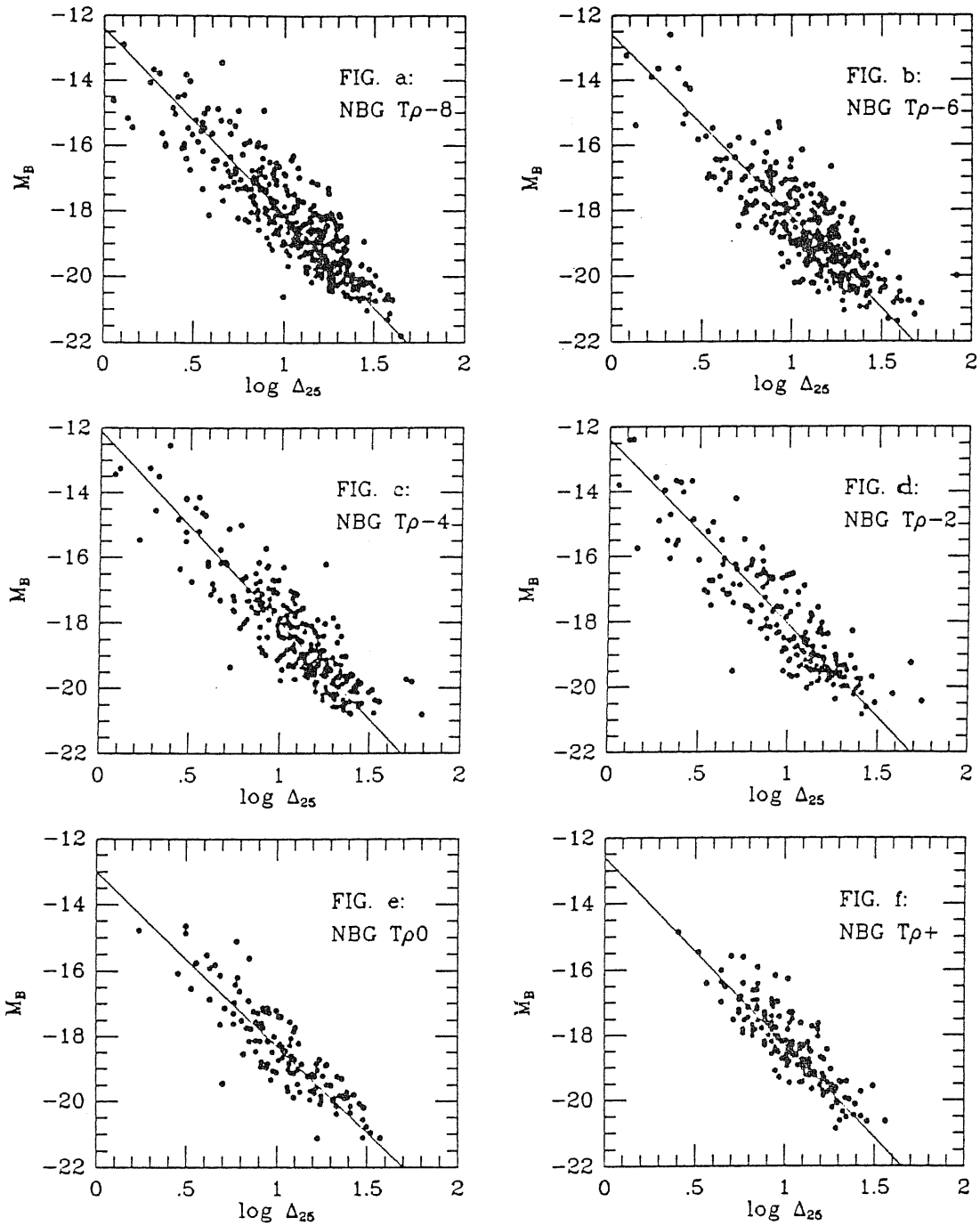

Figure 4.10: a-f) Absolute total blue magnitude vs. logarithm of the isophotal absolute diameter (in kpc): NBG galaxy samples at different mean local densities, with density increasing from Fig.4.10a to Fig.4.10f (see text for further details). The solid lines are the lines fitting the data.

A short discussion is needed for the previous findings concerning the L-D relation for disk galaxies. In particular, I note that my conclusion disagrees with the results obtained by Peterson, Strom, Strom (1979), and Giuricin, Mardirossian, Mezzetti (1985, 1988). The poor sample statistics can be thought to affect the significance of the result obtained by Peterson, Strom, Strom (1978), their sample being composed of ≈ 30 galaxies at most for each of the two clusters considered. Moreover, they did not take into account the data-errors in fitting the L-D relations to their samples, thus underestimating the errors associated with the parameters of the fitted lines. As a consequence, they probably amplified the significance of any existing difference.

On the other hand, Giuricin, Mardirossian, Mezzetti (1988) dealt with a considerable amount of data, yet their samples were taken from a large variety of references, and thus are likely to be quite inhomogeneous. A large inhomogeneity can be responsible for the differences observed; in fact, the results of the previous section made it clear that the inhomogeneity in the L-D relations decreased when samples taken from the same authors were considered. In particular I noted that the "double" samples taken from different authors yielded a lower degree of homogeneity than the "double" samples taken from NBG only, i.e. the comparison of inhomogeneous data samples is likely to increase the inhomogeneity in the L-D relations as well. Moreover, the test used by Giuricin, Mardirossian, Mezzetti (1988), the Welch test, is not well suited for the comparison of more than two samples; the homogeneity test is certainly more appropriate, since it does compare the whole distribution of different values, and not only its tails.

A similar argument can be used to discuss the results obtained by Giuricin, Mardirossian, Mezzetti (1985). They used a single galaxy catalog (i.e., the group catalogue by Geller and Huchra 1983), so that inhomogeneities in the data should be reduced (although the basic parameters, magnitudes and diameters, have been selected from several sources). Nevertheless, their results (regarding the L-D relations for samples of groups with different compacteness) lose significance when the more appropriate homogeneity test is used, instead of the Welch test.

These results are based on a large amount of data for galaxies spanning a wide range of galaxy densities; the statistical tools I used are well suited to the task of comparing many L-D relations. The present data samples are not very homogeneous; nevertheless, the analyses limited to the most homogeneous

samples (e.g., different shells in the same cluster) are in accordance with the overall conclusion. Moreover, any possible inhomogeneity is likely to induce, not to reduce, differences in the L-D relations, whereas I have not found any significant differences in my samples. So, I am led to say that any difference in the L-D relations can be ascribed to differences in the sample selection criteria, and/or to an incorrect choice of the statistical tool used in the comparison analyses.

This result can be restated by saying that the environment has a negligible effect on the L-D relations, either because these are physically unaffected by the local density, or because the scatter in the relations is too large and the accuracy in the data is too low, to allow me to detect any significant deviation from sample to sample. Lastly, I wish to remark that my conclusion on disk galaxies is in line with the recent finding by Giuricin et al. (1989) on elliptical galaxies and the recent theoretical work by Aguilar and White (1986); these results, taken together, seem to constrain previous claims for environmental effects to a lower level of significance.

However, several theorical works (e.g. Faber 1973) suggest that the tidal effect on galaxies is stronger on more external isophotes, so it would be worth extending the previous work studying the L-D relation for D=D_{26} or D_{27} [2]. These data, as the total B-magnitude, are listed in ES0 (now I have an ASCII version of the catalogue) for a large number of galaxies. In ESO, not only it is possible to identify cluster or field members, but the catalogue even reports a density parameter.

Another relevant interesting work would be the study of the L-D relation for the galaxies of compact groups, whose B-magnitude and isophotal radius (for the 24.5 mag arcsec^{-2} isophote) have been recently published (Hickson et al., 1989). In compact groups, if we accept their existence, the very high galaxy density and the strong interactions between galaxies may suggest that there the L-D relation may be affected by the environment.

[2] that is the diameters at 26 B-mag arcsec^{-2} isophote or at 27 B-mag arcsec^{-2} isophote

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