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CP Violation in the Lepton Sector and Implications for Leptogenesis

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Abstract: We review the current status of the data on neutrino masses and lepton mixing and the prospects for measuring the CP-violating phases in the lepton sector. The possible connection between low energy CP violation encoded in the Dirac and Majorana phases of the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix and successful leptogenesis is emphasized in the context of seesaw extensions of the Standard Model with a flavor symmetry G_f (and CP symmetry).

Contents

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1. Introduction: the three-neutrino mixing scheme

The last 19 years witnessed remarkable discoveries in the field of neutrino physics, which point towards new and yet unknown physics beyond the Standard Model (SM). The solar and atmospheric neutrino experiments and the experiments with reactor and accelerator neutrinos have provided irrefutable proofs of neutrino oscillations [1–3] – transitions in flight – between the different flavor neutrinos ν_e, ν_μ, ν_τ (antineutrinos $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) caused by nonzero neutrino masses and lepton mixing (see, e.g., [4] for a review of the relevant data). The experimental discovery of oscillations of atmospheric muon neutrinos and antineutrinos and of the flavor conversion of solar (electron) neutrinos led to the 2015 Nobel Prize for Physics awarded to Takaaki Kajita [5] (from the SuperKamiokande Collaboration) and Arthur B. McDonald [6] (from the SNO Collaboration). The existence of flavor neutrino oscillations implies the presence of mixing in the lepton weak charged current (CC), namely

$$\mathcal{L}_{\rm CC} = -\,\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \bar{e}_{L\alpha}\,\gamma_\mu \nu_{L\alpha}\,W^{\mu\dagger} + \text{h.c.}\,, \qquad \nu_{L\alpha} = \sum_{j=1}^{n} (U_{\rm PMNS})_{\alpha j}\;\nu_{jL}\,, \quad (1)$$

where $\nu_{L\alpha}$ is the flavor neutrino field, ν_{jL} is the left-handed (LH) component of the field of the neutrino ν_j having a mass m_j, and $U_{\rm PMNS}$ is a unitary matrix – the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [1–3], $U_{\rm PMNS} \equiv U_\nu$.[a]

All well established statistically significant data on neutrino oscillations can be described within the three-neutrino mixing scheme, $n = 3$. The number of neutrinos with definite mass can, in general, be bigger than three if, e.g., there exist right-handed (RH) neutrinos [3] and they mix effectively (via Majorana and Dirac mass terms) with the LH flavor neutrinos. It follows from the current data that at least three of the neutrinos with definite mass ν_j, say ν_1, ν_2, ν_3, must be light, with $m_{1,2,3} \lesssim 1$ eV, and must have different masses, $m_1 \neq m_2 \neq m_3$. In what concerns the masses of the possible additional mass-eigenstate neutrinos, ν_k, $k = 4, 5, \ldots$, there are a number of different possibilities and we refer the reader to the discussions in other chapters of this review [7–9].[b]

In the case of three light neutrinos, $n = 3$, the three-by-three unitary lepton mixing matrix U_ν can be parametrized, as is well known, by three angles and one Dirac phase for Dirac neutrinos, and with two additional Majorana phases in case neutrinos are Majorana particles [15], that is

$$U_\nu = \hat{U}_\nu \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta})\,, \quad (2)$$

[a] We work here in a basis where all the lepton mixing originates from the neutrino sector, unless stated otherwise.

[b] At present there are several experimental inconclusive indications for the existence of one or two light RH neutrinos at the eV scale, which mix with the flavor neutrinos, implying the presence of one or two additional neutrinos, ν_4 or $\nu_{4,5}$, with masses m_4 ($m_{4,5}$) ~ 1 eV (see, e.g., [4]). For a discussion of the current status of these indications, of their upcoming tests and of the related implications see, e.g., [10–14].

where α and β are the two Majorana phases, $\alpha, \beta \in [0, \pi)$, and

$$\hat{U}_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \tag{3}$$

In Eq. (3), $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} \in [0, \pi/2)$, and $\delta \in [0, 2\pi)$ is the Dirac phase. Thus, if ν_j are Dirac particles, the PMNS mixing matrix is similar, in both the number of mixing angles and CP phases, to the quark mixing matrix. U_ν contains two additional physical CP phases, if ν_j are Majorana fermions due to the special properties of Majorana particles (see, e.g., [15–17]). On the basis of the currently existing neutrino data it is impossible to determine the nature – Dirac or Majorana – of the neutrinos with definite mass ν_j.

The neutrino oscillation probabilities are functions of the neutrino energy, E, of the source-detector distance L, of the elements of U_ν and, for relativistic neutrinos used in all neutrino experiments performed so far, of the neutrino mass squared differences $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, $i \neq j$ (see, e.g., [4, 16]). In the three-neutrino mixing scheme, there are only two independent Δm^2_{ij}, say $\Delta m^2_{21} \neq 0$ and $\Delta m^2_{31} \neq 0$. In the widely used convention of numbering the neutrinos with definite mass ν_j that we are going to employ, θ_{12}, $\Delta m^2_{21} > 0$, and θ_{23}, Δm^2_{31}, represent the parameters which drive the solar (ν_e) and the dominant atmospheric (ν_μ and $\bar{\nu}_\mu$) oscillations, respectively, and θ_{13} is associated with the smallest mixing angle in the PMNS mixing matrix. The existing data allows a determination of θ_{12}, Δm^2_{21}, and θ_{23}, $|\Delta m^2_{31(32)}|$ and θ_{13}, with a few percent precision [18–20]. The best fit value (bfv) and the $3\,\sigma$ allowed ranges of Δm^2_{21}, $\sin^2\theta_{12}$, $|\Delta m^2_{31(32)}|$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ read [18]

$$\begin{aligned} &(\Delta m^2_{21})_{\rm bfv} = 7.37 \times 10^{-5}\ \ {\rm eV}^2, \qquad \Delta m^2_{21} = (6.93 \div 7.96) \times 10^{-5}\ \ {\rm eV}^2\,, \\ &(\sin^2\theta_{12})_{\rm bfv} = 0.297, \qquad 0.250 \leq \sin^2\theta_{12} \leq 0.354\,, \\ &(|\Delta m^2_{31(32)}|)_{\rm bfv} = 2.56\ (2.54) \times 10^{-3}\ \ {\rm eV}^2\,, \\ &|\Delta m^2_{31(32)}| = (2.45\ (2.42) \div 2.69\ (2.66)) \times 10^{-3}\ \ {\rm eV}^2\,, \\ &(\sin^2\theta_{23})_{\rm bfv} = 0.425\ (0.589)\,, \quad 0.381\,(0.384) \leq \sin^2\theta_{23} \leq 0.615\,(0.636)\,, \\ &(\sin^2\theta_{13})_{\rm bfv} = 0.0215\ (0.0216)\,, \quad 0.0190\,(0.0190) \leq \sin^2\theta_{13} \leq 0.0240\,(0.0242)\,, \end{aligned} \tag{4}$$

where the value (the value in brackets) corresponds to $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). The current data does not fix the sign of $\Delta m^2_{31(32)}$. It follows from the results quoted above that $\Delta m^2_{21}/|\Delta m^2_{31(32)}| \approx 0.03$ and $|\Delta m^2_{31}| = |\Delta m^2_{32} + \Delta m^2_{21}| \approx |\Delta m^2_{32}|$. Notice also that the atmospheric mixing angle can be maximal, $\theta_{23} = \pi/4$, within the $3\,\sigma$ interval. In contrast, the value of $\theta_{12} = \pi/4$, i.e., maximal solar neutrino mixing, is definitely ruled out by the data. Thus, one has $\theta_{12} < \pi/4$ and at 99.73% CL, $\cos 2\theta_{12} \geq 0.29$. The quoted results imply also that the pattern of lepton mixing differs drastically from the pattern of quark mixing.

Starting from 2013 there are also persistent hints in the neutrino oscillation data that the value of the Dirac phase $\delta \approx 3\pi/2$ (see, e.g., [21, 22]).[c] In the two recent analyses [18, 19] the authors find for the bfv of δ, respectively, $\delta = 1.38\pi$ ($\delta = 1.31\pi$) and $\delta = 1.45\pi$ ($\delta = 1.54\pi$) in the case of $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$). According to the results in [18] the CP-conserving value $\delta = 0$ is disfavored at $2.4\,\sigma$ (at $3.2\,\sigma$) for $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$), while the maximal CP-violating value $\delta = \pi/2$ is ruled out at $3.4\,\sigma$ (at $3.9\,\sigma$). The second CP-conserving value $\delta = \pi$ is statistically about $2.0\,\sigma$ away from the bfv $\delta = 1.38\pi$ (see, e.g., figure 1 in [18]). The hint that $\delta \approx 3\pi/2$ is strengthened by the results of the NOνA neutrino oscillation experiment [26, 27]. The relatively large value of $\sin\theta_{13} \approx 0.15$, measured in the Daya Bay [28–31], RENO [32–34] and Double Chooz [35–37] experiments, combined with the value of $\delta \approx 3\pi/2$ has far-reaching implications for the searches for CP violation in neutrino oscillations (see discussion in Sec. 2).

As we have noted, the sign of $\Delta m^2_{31(32)}$ cannot be determined from the existing data. In the three-neutrino mixing scheme, the two possible signs of $\Delta m^2_{31(32)}$ correspond to two types of neutrino mass spectrum. In the convention of numbering of neutrinos ν_j employed by us, the two spectra read:
i) spectrum with normal ordering (NO): $m_1 < m_2 < m_3$, i.e., $\Delta m^2_{31(32)} > 0$ and $\Delta m^2_{21} > 0$;
ii) spectrum with inverted ordering (IO): $m_3 < m_1 < m_2$, i.e., $\Delta m^2_{32(31)} < 0$, $\Delta m^2_{21} > 0$, $m_2 = \sqrt{m_3^2 + |\Delta m^2_{32}|}$ and $m_1 = \sqrt{m_3^2 + |\Delta m^2_{32}| - \Delta m^2_{21}}$.
Depending on the values of the lightest neutrino mass, the neutrino mass spectrum can be further distinguished:
a) normal hierarchical (NH): $m_1 \ll m_2 < m_3$, $m_2 \approx \sqrt{\Delta m^2_{21}} \approx 8.6 \times 10^{-3}$ eV, $m_3 \approx \sqrt{\Delta m^2_{31}} \approx 0.051$ eV;
b) inverted hierarchical (IH): $m_3 \ll m_1 < m_2$, $m_1 \approx \sqrt{|\Delta m^2_{32}| - \Delta m^2_{21}} \approx 0.050$ eV, $m_2 \approx \sqrt{|\Delta m^2_{32}|} \approx 0.051$ eV;
c) quasi-degenerate (QD): $m_1 \approx m_2 \approx m_3 \approx m_{\rm QD}$, $m_j^2 \gg |\Delta m^2_{31(32)}|$, $m_{\rm QD} \gtrsim 0.10$ eV.
All three types of spectra are compatible with the constraints on the absolute scale of neutrino masses. Determining the type of neutrino mass spectrum is one of the main goals of the future experiments in the field of neutrino physics (see, e.g., [4, 38]).[d]

Data on the absolute neutrino mass scale can be obtained, e.g., from measurements of the spectrum of electrons near the endpoint in ^{3}H β-decay experi-

[c]Using the T2K data on $\nu_\mu \to \nu_e$ oscillations, the T2K Collaboration [23, 24] found in 2013 that for $\delta = 0$, $\sin^2\theta_{23} = 0.5$ and $|\Delta m^2_{31(32)}| = 2.4 \times 10^{-3}$eV^2, in the case of $\Delta m^2_{31(32)} > 0$ ($\Delta m^2_{31(32)} < 0$), $\sin^2 2\theta_{13} = 0.140^{+0.038}_{-0.032}$ ($0.170^{+0.045}_{-0.037}$). Thus, the bfv of $\sin^2 2\theta_{13}$ obtained in the T2K experiment for $\delta = 0$ was approximately a factor of 1.6 (1.9) bigger than that measured in the Daya Bay experiment [25]: $\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$. The compatibility of the results of the two experiments on $\sin^2 2\theta_{13}$ required, in particular, that $\delta \neq 0$, which led to the hint that $\delta \approx 3\pi/2$ in the analyses [21–23] of the neutrino oscillation data.
[d]For a review of the experiments which can provide data on the type of neutrino mass spectrum see, e.g., [38]. For some specific proposals see, e.g., [39–41].

ments [42–46] and from cosmological and astrophysical observations [47]. The most stringent upper bound on the $\bar{\nu}_e$ mass was reported by the Troitzk experiment [46]

$$m_{\bar{\nu}_e} < 2.05 \text{ eV} \quad \text{at} \quad 95\% \quad \text{CL}\,.$$

A similar result was obtained in the Mainz experiment [44]: $m_{\bar{\nu}_e} < 2.3$ eV at 95% CL. We have $m_{\bar{\nu}_e} \approx m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment [45, 48] is designed to reach a sensitivity of $m_{\bar{\nu}_e} = 0.20$ eV, i.e., to probe the region of the QD spectrum.

Constraints on the sum of the neutrino masses can be obtained from cosmological and astrophysical data [47]. Depending on the complexity of the underlying model and the input data used one typically obtains

$$\sum_j m_j \lesssim (0.3 \div 1.3) \text{ eV} \quad \text{at} \quad 95\% \quad \text{CL}\,. \tag{5}$$

Assuming the existence of three light massive neutrinos and the validity of the Λ CDM (Cold Dark Matter) model, and using data on the Cosmic Microwave Background (CMB) temperature power spectrum anisotropies, on gravitational lensing effects and the low ℓ CMB polarization spectrum data (the "low P" data), the Planck Collaboration [49] reported the updated upper limit: $\sum_j m_j < 0.57$ eV, at 95% CL. Adding supernovae (SN) light-curve data and data on the Baryon Acoustic Oscillations (BAO) strengthens the limit to [e]

$$\sum_j m_j < 0.153 \text{ eV} \quad \text{at} \quad 95\% \quad \text{CL}\,. \tag{6}$$

Understanding the origin of the observed pattern of lepton mixing, establishing the status of the CP symmetry in the lepton sector, determining the type of spectrum the neutrino masses obey and determining the nature – Dirac or Majorana – of massive neutrinos, are among the highest priority goals of the program of future research in neutrino physics (see, e.g., [4, 50–59]). The principal goal is to reveal at a fundamental level the mechanism giving rise to neutrino masses and lepton mixing.

2. Observables related to low energy CP violation in the lepton sector

Apart from the hint that the Dirac phase $\delta \approx 3\pi/2$, no other experimental information on the CP phases in the lepton sector is available at present. Thus, the status of CP symmetry in the lepton sector is essentially unknown. The determination of the Dirac and Majorana phases might shed further light on the organizing principles in the lepton (and possibly the quark) sector(s) (see discussion in Sec. 4).

[e]Result quoted in wiki.cosmos.esa.int/planckpla2015, page 311.

2.1. *Dirac CP violation*

Taking into account that the reactor mixing angle is not so small, $\theta_{13} \approx 0.15$, a non-zero value of the Dirac phase δ can generate CP-violating effects in neutrino oscillations [15, 60, 61], i.e., a difference between the probabilities of the $\nu_\alpha \to \nu_\beta$ and $\bar{\nu}_\alpha \to \bar{\nu}_\beta$ oscillations, $\alpha \neq \beta$. A measure of CP violation is provided, e.g., by the asymmetries

$$A_{\rm CP}^{(\alpha,\beta)} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)\,, \quad \alpha \neq \beta \quad \text{and} \quad \alpha,\, \beta = e, \mu, \tau\,. \tag{7}$$

The magnitude of CP-violating effects in neutrino oscillations in the three-neutrino mixing scheme is controlled, as is well known [62], by the rephasing invariant $J_{\rm CP}$ associated with the Dirac phase δ,

$$A_{\rm CP}^{(e,\mu)} = A_{\rm CP}^{(\mu,\tau)} = -A_{\rm CP}^{(e,\tau)} = J_{\rm CP}\; F_{\rm osc}^{\rm vac}\,, \tag{8}$$

with

$$\begin{aligned} J_{\rm CP} &= {\rm Im}\left((U_\nu)_{e2}\, (U_\nu)^*_{\mu 2}\, (U_\nu)^*_{e3}\, (U_\nu)_{\mu 3} \right)\,, \\ F_{\rm osc}^{\rm vac} &= \sin\left(\frac{\Delta m^2_{21} L}{2E}\right) + \sin\left(\frac{\Delta m^2_{32} L}{2E}\right) + \sin\left(\frac{\Delta m^2_{13} L}{2E}\right)\,. \end{aligned} \tag{9}$$

The factor $J_{\rm CP}$ in the expressions for $A_{\rm CP}^{(\alpha,\beta)}$ is analogous to the rephasing invariant associated with the Dirac phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, introduced in [63]. In the standard parametrization of the PMNS mixing matrix given in Eq. (2) and Eq. (3), $J_{\rm CP}$ has the form

$$J_{\rm CP} = \frac{1}{8}\, \cos\theta_{13} \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin 2\theta_{13}\, \sin\delta\,. \tag{10}$$

As we discussed, the existing neutrino oscillation data has allowed the determination of the mixing angles θ_{12}, θ_{23} and θ_{13} with a few to several percent precision. The size of CP-violating effects in neutrino oscillations is still unknown, because the precise value of the Dirac phase δ is currently unknown. The current data implies that in the case of NO (IO) neutrino mass spectrum

$$0.026\ (0.027)\, |\sin\delta| \lesssim |J_{\rm CP}| \lesssim 0.036\, |\sin\delta|\,, \tag{11}$$

where we have used the $3\,\sigma$ ranges of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ given in Eq. (4). For the bfv of $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ and δ found in [18] we get for $\Delta m^2_{31(2)} > 0$ ($\Delta m^2_{31(2)} < 0$): $J_{\rm CP} \approx -\,0.031$ ($J_{\rm CP} \approx -\,0.026$). Thus, if the indication that $\delta \approx 3\pi/2$ is confirmed by future more precise data, the CP-violating effects in

neutrino oscillations would be relatively large, provided the factor $F^{\rm vac}_{\rm osc}$ does not lead to a suppression of the asymmetries $A^{(\alpha,\beta)}_{\rm CP}$. Such a suppression would not occur if, under the conditions of a given experiment, both neutrino mass squared differences Δm^2_{21} and $\Delta m^2_{31(32)}$ are "operative", i.e., if neither $\sin(\Delta m^2_{21}L/(2E)) \approx 0$ nor $\sin(\Delta m^2_{31(32)}L/(2E)) \approx 0$.

The search for CP-violating effects from the Dirac phase in neutrino oscillations is one of the principal goals of the future experimental studies in neutrino physics (see, e.g., [24, 26, 27, 51–55, 64]). In order for the CP-violating effects in neutrino oscillations to be observable, both $\sin(\Delta m^2_{31}L/(2E))$ and $\sin(\Delta m^2_{21}L/(2E))$ should be sufficiently large. In the case of $\sin(\Delta m^2_{31}L/(2E))$, for instance, this requires that, say, $\Delta m^2_{31}L/(2E) \approx 1$. The future experiments on CP violation in neutrino oscillations are planned to be performed with accelerator ν_μ and $\bar{\nu}_\mu$ beams with energies from ~ 0.7 GeV to a few GeV. Taking as an instructive example $E = 1$ GeV and using the bfv of $\Delta m^2_{31} = 2.56 \times 10^{-3}$ eV^2, one has $\Delta m^2_{31}L/(2E) \approx 1$ for $L \approx 1000$ km. Thus, the possibility to observe CP violation in neutrino oscillations requires the experiments to have long baselines. The MINOS, T2K and NOνA experiments, for example, which provide data on ν_μ oscillations (see, e.g., [4] and references therein), have baselines of approximately 735 km, 295 km and 810 km, respectively. The planned DUNE [51–53] and T2HK [54, 55, 65] experiments, which are designed to search for CP violation effects in neutrino oscillations, will have baselines of 1300 km and 295 km, respectively.

Thus, in MINOS, T2K, NOνA and in the future planned experiments DUNE and T2HK, the baselines are such that the neutrinos travel relatively long distances in the matter of the Earth mantle. As is well known, the pattern of neutrino oscillations can be changed significantly by the presence of matter [66, 67] (see also [68]) due to the coherent (forward) scattering of neutrinos on the "background" of electrons (e^-), protons (p) and neutrons (n) present in matter. The scattering generates an effective potential $V_{\rm eff}$ in the neutrino Hamiltonian: $H = H_{\rm vac} + V_{\rm eff}$, which modifies the vacuum lepton mixing angles, since the eigenstates and eigenvalues of $H_{\rm vac}$ and of H differ. This leads to different oscillation probabilities compared with oscillations in vacuum. The matter in the Earth (and the Sun) is not charge conjugation (C-) symmetric, since it contains only e^-, p and n but does not contain their antiparticles. As a consequence, the oscillations taking place in the Earth are neither CP- nor CPT-invariant [69]. This complicates the studies of CP violation due to the Dirac phase δ in long baseline neutrino oscillation experiments.

In the constant density approximation and keeping terms up to second order in the two small parameters $|\Delta m^2_{21}/\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$, the expression for the $\nu_\mu \to \nu_e$ oscillation probability in the three-neutrino mixing scheme and for neutrinos crossing the Earth mantle, has the form [70] (see also [71])

$$P^{3\nu\ \rm man}_m(\nu_\mu \to \nu_e) \approx P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,. \tag{12}$$

In the previous expression we define

$$
\begin{aligned}
P_0 &= \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta]\,,\\
P_3 &= r^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta)\,,\\
P_{\sin\delta} &= -\,r\,\frac{8\,J_{\rm CP}}{A(1-A)}\,\sin\Delta\,\sin(A\Delta)\,\sin\left[(1-A)\Delta\right]\,,\\
P_{\cos\delta} &= r\,\frac{8\,J_{\rm CP}\,\cot\delta}{A(1-A)}\,\cos\Delta\,\sin(A\Delta)\,\sin\left[(1-A)\Delta\right]\,,
\end{aligned}
\tag{13}
$$

where

$$
r = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\,,\qquad \Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\qquad A = \sqrt{2}\,G_{\rm F}\,N_e^{\rm man}\,\frac{2E}{\Delta m^2_{31}}\,,
\tag{14}
$$

$G_{\rm F}$ is the Fermi constant and $N_e^{\rm man}$ denotes the electron number density of the Earth mantle. The Earth matter effects on the oscillations are accounted for by the quantity A. The mean electron number density in the Earth mantle is $\bar{N}_e^{\rm man} \approx 1.5\ {\rm cm}^{-3}\ N_A$ [72], N_A being Avogadro's number. $N_e^{\rm man}$ varies little around the indicated mean value along the trajectories of the neutrinos in the Earth mantle corresponding to the experiments under discussion. Thus, as far as the calculation of neutrino oscillation probabilities is concerned, the constant density approximation $N_e^{\rm man} = \tilde{N}_e^{\rm man}$, where $\tilde{N}_e^{\rm man}$ is the (constant) mean density along the given neutrino path in the Earth, was shown to be sufficiently accurate [62, 73, 74]. The expression for the probability of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation can be obtained formally from that for $P_m^{3\nu\ {\rm man}}(\nu_\mu \to \nu_e)$ by making the changes $A \to -A$ and $J_{\rm CP} \to -J_{\rm CP}$, with $J_{\rm CP}\cot\delta$ remaining unchanged.[f] If the Dirac phase in the PMNS mixing matrix U_ν has a CP-conserving value, we would have $P_{\sin\delta} = 0$. However, we would still have that $P_m^{3\nu\ {\rm man}}(\nu_\mu \to \nu_e)$ and $P_m^{3\nu\ {\rm man}}(\bar{\nu}_\mu \to \bar{\nu}_e)$ are unequal due to the Earth matter effects. It is possible, in principle, to experimentally disentangle the effects of the Earth matter and of $J_{\rm CP}$ in the asymmetries for neutrinos crossing the Earth mantle, $A_{\rm CP}^{(e,\mu)\ {\rm man}}$, by studying the energy dependence of $P_m^{3\nu\ {\rm man}}(\nu_\mu \to \nu_e)$ and $P_m^{3\nu\ {\rm man}}(\bar{\nu}_\mu \to \bar{\nu}_e)$. This will allow to obtain direct information about Dirac CP violation in the lepton sector and to measure the Dirac phase δ. In the vacuum limit $N_e^{\rm man} = 0$ $(A = 0)$ we have $A_{\rm CP}^{(e,\mu)\ {\rm man}} = A_{\rm CP}^{(e,\mu)}$, see Eq. (8), and only the term $P_{\sin\delta}$ contributes to $A_{\rm CP}^{(e,\mu)}$.

The expressions for the probabilities $P_m^{3\nu\ {\rm man}}(\nu_\mu \to \nu_e)$ and $P_m^{3\nu\ {\rm man}}(\bar{\nu}_\mu \to \bar{\nu}_e)$ can be used in the interpretation of the results of MINOS, T2K, NOνA, and of the future planned DUNE and T2HK experiments. For a discussion of the sensitivity of these experiments to δ see, e.g., [24, 26, 27, 51–55, 64]. If indeed $\delta \approx 3\pi/2$, the DUNE and T2HK experiments are foreseen to establish the existence of leptonic Dirac CP violation at $\sim 5\,\sigma$.

[f] For a detailed discussion of the conditions of validity of the analytic expression for $P_m^{3\nu\ {\rm man}}(\nu_\mu \to \nu_e)$ quoted above see [70].

2.2. *Majorana phases and neutrinoless double beta decay*

If neutrinos are massive Majorana particles, the lepton mixing matrix contains two additional Majorana phases [15], α and β, see Eq. (2). However, getting experimental information about these CP-violating phases is a remarkably difficult problem [75–79]. In fact, the flavor neutrino oscillation probabilities are insensitive to the CP phases α and β. The Majorana phases play an important role in processes characteristic of the Majorana nature of massive neutrinos.

The massive Majorana neutrinos can mediate processes in which the total lepton number changes by two units, $|\Delta L| = 2$, such as $K^+ \to \pi^- + \mu^+ + \mu^+$ and $e^- + (A, Z) \to e^+ + (A, Z-2)$. The rates of these processes are typically proportional to the factor $(m_j/M(|\Delta L| = 2))^2$, $M(|\Delta L| = 2)$ being the characteristic mass scale of the given process, and thus are typically extremely small. The experimental searches for neutrinoless double beta ($0\nu\beta\beta$) decay, $(A, Z) \to (A, Z+2) + e^- + e^-$, of even-even nuclei, e.g., ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd, are unique in reaching a sensitivity that might allow to observe this process, if it is triggered by the exchange of the light neutrinos ν_j or new physics beyond the SM (see, e.g., [17, 50, 80, 81]). The half-life $T^{0\nu}_{1/2}$ of an isotope decaying via this process, induced by the exchange of virtual $\nu_{1,2,3}$, takes the form (see, e.g., [82])

$$\frac{1}{T^{0\nu}_{1/2}} \;=\; G^{0\nu}\,|\mathcal{M}(A,Z)|^2\,\frac{|m_{\beta\beta}|^2}{m_e}\,, \tag{15}$$

where $\mathcal{M}(A, Z)$ denotes the nuclear matrix element (NME) for a $|\Delta L| = 2$ transition, $G^{0\nu}$ is a phase space factor, m_e the electron mass and $|m_{\beta\beta}|$ the $0\nu\beta\beta$ decay effective Majorana mass. The latter contains all the dependence of $T^{0\nu}_{1/2}$ on the lepton mixing parameters. We have

$$|m_{\beta\beta}| = \left| m_1\, c^2_{12}\, c^2_{13} \;+\; m_2\, s^2_{12}\, c^2_{13}\, e^{2i\alpha} \;+\; m_3\, s^2_{13}\, e^{2i\beta'} \right|\,, \tag{16}$$

with $\beta' \equiv \beta - \delta$. For the NH, IH and QD neutrino mass spectra, $|m_{\beta\beta}|$ is given by

$$\begin{aligned}
\text{NH}\,, \quad |m_{\beta\beta}| \;&\approx\; \left| \sqrt{\Delta m^2_{21}}\; s^2_{12} + \sqrt{\Delta m^2_{31}}\; s^2_{13} e^{2i(\beta'-\alpha)} \right|\,, \\
\text{IH}\,, \quad |m_{\beta\beta}| \;&\approx\; \sqrt{|\Delta m^2_{32}|}\;\left| c^2_{12} + s^2_{12}\; e^{2i\alpha} \right|\,, \\
\text{QD}\,, \quad |m_{\beta\beta}| \;&\approx\; m_{\text{QD}}\;\left| c^2_{12} + s^2_{12}\; e^{2i\alpha} \right|\,.
\end{aligned} \tag{17}$$

Obviously, $|m_{\beta\beta}|$ strongly depends on the Majorana phases: the CP-conserving values of α=$0, \pi/2$ [83–85], for instance, determine the range of possible values of $|m_{\beta\beta}|$ in the cases of IH and QD spectrum.

Using the $3\,\sigma$ ranges of the allowed values of the neutrino oscillation parameters quoted in Eq. (4), one finds that:
i) 0.78×10^{-3} eV $\lesssim\; |m_{\beta\beta}| \;\lesssim 4.32 \times 10^{-3}$ eV in the case of NH spectrum;

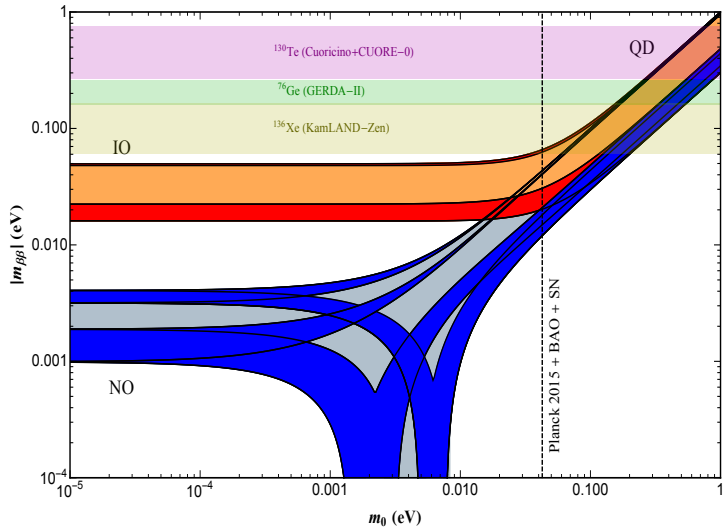

Fig. 1. The effective Majorana mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass m_0. The figure is obtained using the bfvs and the $2\,\sigma$ ranges of allowed values of Δm^2_{21}, $\sin^2\theta_{12}$, and $|\Delta m^2_{31}| \approx |\Delta m^2_{32}|$, $\sin^2\theta_{13}$ from [21]. The CP phases α and β' are varied in the interval $[0,\pi)$. The predictions for the NO, IO and QD mass spectra are indicated. In the light-blue and orange regions at least one of the phases α, β' and $(\beta' - \alpha)$ has a CP-violating value, while the blue and red areas correspond to α and β' possessing CP-conserving values. The 90% CL exclusion limits on $|m_{\beta\beta}|$ from searches for $0\nu\beta\beta$ decay and the 95% CL upper bound on m_0 from cosmological data are indicated.

ii) 1.4×10^{-2} eV $\lesssim\ |m_{\beta\beta}|\ \lesssim 5.1 \times 10^{-2}$ eV in the case of IH spectrum;
iii) $m_{\rm QD}/3 \lesssim\ |m_{\beta\beta}|\ \lesssim m_{\rm QD}$, $m_{\rm QD} \gtrsim 0.10$ eV, in the case of QD spectrum.
The difference in the ranges of $|m_{\beta\beta}|$ in these three cases opens up the possibility to get information about the type of neutrino mass spectrum from a measurement of $|m_{\beta\beta}|$ [40]. The main features of the predictions for $|m_{\beta\beta}|$ are illustrated in Fig. 1, where $|m_{\beta\beta}|$ is shown as a function of the lightest neutrino mass m_0.

The predictions for $|m_{\beta\beta}|$ in the cases of the NO, IO and QD neutrino mass spectra can be drastically modified by the existence of lepton number violating $|\Delta L| = 2$ new physics beyond that predicted by the SM: eV or GeV to TeV scale RH neutrinos, etc. (see, e.g., [86–92]). There is a potential synergy between the searches for $0\nu\beta\beta$ decay and the searches for neutrino-related $|\Delta L| = 2$ beyond the SM physics at LHC [92].

The best lower limit on the half-life of ^{76}Ge, $T^{0\nu}_{1/2}(^{76}\text{Ge}) > 5.2 \times 10^{25}$ yr (at 90% CL), was found in the GERDA-II ^{76}Ge experiment [93]. This lower limit on $T^{0\nu}_{1/2}(^{76}\text{Ge})$ corresponds to the upper limit $|m_{\beta\beta}| < (0.16 \div 0.26)$ eV, in which the estimated uncertainties in the NME are accounted for. Two experiments, NEMO-3 [94] with ^{100}Mo and CUORICINO+CUORE-0 [95] with ^{130}Te, obtained the limits (at 90% CL): $T^{0\nu}_{1/2}(^{100}\text{Mo}) > 1.1 \times 10^{24}$ yr corresponding to $|m_{\beta\beta}| < (0.3 \div 0.6)$ eV [94] with the estimated NME uncertainties taken into account, and $T^{0\nu}_{1/2}(^{130}\text{Te}) >$

4.4×10^{24} yr [95]. The best lower limit on the $0\nu\beta\beta$ decay half-life of ^{136}Xe was obtained by the KamLAND-Zen Collaboration [96]: $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.07 \times 10^{26}$ yr, corresponding to $|m_{\beta\beta}| < (0.061 \div 0.165)$ eV (at 90% CL). The EXO Collaboration [97] reported the lower limit: $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.6 \times 10^{25}$ yr (at 90% CL).

Most importantly, a large number of experiments of a new generation aim at sensitivities to $|m_{\beta\beta}| \sim (0.01 \div 0.05)$ eV (see, e.g., [50]): CUORE (^{130}Te), SNO+ (^{130}Te), GERDA (^{76}Ge), MAJORANA (^{76}Ge), LEGEND (^{76}Ge), SuperNEMO (^{82}Se, ^{150}Nd), KamLAND-Zen (^{136}Xe), EXO and nEXO (^{136}Xe), PANDAX-III (^{136}Xe), NEXT (^{136}Xe), AMoRE (^{100}Mo), MOON (^{100}Mo), CANDLES (^{48}Ca), XMASS (^{136}Xe), DCBA (^{82}Se, ^{150}Nd), ZICOS (^{96}Zr), etc. The GERDA-II and KamLAND-Zen experiments have already provided the best lower limits on the $0\nu\beta\beta$ decay half-lives of ^{76}Ge and ^{136}Xe. The experiments listed above aim to probe the QD and IO ranges of $|m_{\beta\beta}|$. If $0\nu\beta\beta$ decay will be observed in these experiments, the measurement of the $0\nu\beta\beta$ decay half-life might allow to obtain constraints on the Majorana phase α, see [75, 76, 98, 99].

Proving that the CP symmetry is violated in the lepton sector due to the Majorana phases α and $\beta^{(\prime)}$ is remarkably challenging [76–78]: it requires quite accurate measurements of $|m_{\beta\beta}|$ (and of m_{QD} for QD spectrum), and holds only for a limited range of values of the relevant parameters. For $\sin^2\theta_{12}$=0.31 in the case of QD spectrum, for example, establishing at 95% CL that the relevant phase α possesses a CP-violating value requires [78] a relative error on the measured value of $|m_{\beta\beta}|$ and m_0 smaller than 15%, a theoretical uncertainty $F \lesssim 1.5$ in the corresponding NME, and a value of α typically within the ranges of $(\pi/8 \div 3\pi/8)$ and $(5\pi/8 \div 7\pi/8)$.

Obtaining quantitative information on the lepton mixing parameters from a measurement of $0\nu\beta\beta$ decay half-life would be impossible without sufficiently precise knowledge of the corresponding NME of the process.[g] At present the variation of the values of the different $0\nu\beta\beta$ decay NMEs calculated using the various currently employed methods is typically by a factor of two to three. The observation of $0\nu\beta\beta$ decay of one nucleus is likely to lead to the searches and observation of the decay of other nuclei. The data on the $0\nu\beta\beta$ decay of several nuclei might help to solve the problem of sufficiently precise calculation of the $0\nu\beta\beta$ decay NMEs [76].

An additional source of uncertainty is the effective value of the axial-vector coupling constant g_A in $0\nu\beta\beta$ decay. This constant is renormalized by nuclear medium effects, which tend to quench, i.e., reduce, the vacuum value of g_A. The problem of the quenching of g_A arose in connection with the efforts to describe theoretically the experimental data on the two-neutrino double beta decay. The physical origin of the quenching is not fully understood, and the size of the quenching of g_A in $0\nu\beta\beta$ decay is subject to debates (for further details see, e.g., [82]). The importance of the effective value of g_A in $0\nu\beta\beta$ decay stems from the fact that, to a good approximation, the $0\nu\beta\beta$ decay rate is proportional to the fourth power of

[g]For discussions of the current status of the calculations of the NMEs for $0\nu\beta\beta$ decay see, e.g., [82]. A possible test of the NME calculations is suggested in [76] and is discussed in greater detail in [100].

the effective g_A.

If the future $0\nu\beta\beta$ decay experiments show that $|m_{\beta\beta}| < 0.01$ eV, both the IO and the QD spectrum will be ruled out for massive Majorana neutrinos. If in addition it is established in neutrino oscillation experiments that $\Delta m^2_{31(32)} < 0$ (IO spectrum), one would be led to conclude that either the massive neutrinos ν_j are Dirac fermions, or that ν_j are Majorana particles, but there are additional contributions to the $0\nu\beta\beta$ decay amplitude which interfere destructively with that due to the exchange of ν_j. The case of more than one mechanism generating the $0\nu\beta\beta$ decay is discussed in, e.g., [101, 102], where the possibility to identify the mechanisms inducing the decay is also analyzed. If, however, $\Delta m^2_{31(32)}$ is determined to be positive, the upper limit $|m_{\beta\beta}| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing NH mass spectrum, or NO spectrum with partial hierarchy, and the quest for $|m_{\beta\beta}|$ would still be open [103].

3. Type I seesaw mechanism of neutrino mass generation and leptogenesis

It follows from the neutrino data summarized in the previous sections that neutrino masses are much smaller than the masses of the other SM fermions. If we take as an indicative upper limit $m_i \lesssim 0.5$ eV, we have $m_i/m_e \lesssim 10^{-6}$. It is natural to suppose that the remarkable smallness of neutrino masses is related to the existence of a new fundamental mass scale in particle physics, and thus to new physics beyond that predicted by the SM.

A possible explanation of neutrino masses is provided by the seesaw mechanism of neutrino mass generation. In the type I seesaw realization [104–107], the SM is extended with at least two heavy RH neutrinos N_a, with $P_R N_a = N_a$, coupled to the SM leptons via the seesaw Lagrangian

$$\mathcal{L} \;=\; \mathcal{L}_{\rm SM} + \bar{N}_a\, i\partial\!\!\!/\, N_a - \left(\frac{1}{2}\,(M_N)_{ab}\,\bar{N}^c_a N_b + \lambda_{\alpha b}\,\bar{\ell}_\alpha \phi^c N_b + \text{h.c.}\right), \qquad (18)$$

where $\ell_\alpha \equiv (\nu_{L\alpha},\, e_{L\alpha})^T$ is the SM lepton doublet of a given flavor $\alpha = e, \mu, \tau$, $\phi^c \equiv \epsilon\,\phi^*$ ($\epsilon_{12} = -1$) is the conjugate Higgs doublet and $N^c_a \equiv C\bar{N}^T_a$, with $P_L N^c_a = N^c_a$. We denote by $\mathcal{N}_k = (U_R)^*_{ak} N_a$ the RH neutrino fields with definite masses $M_k > 0$, U_R being the unitary matrix which diagonalizes the Majorana mass matrix M_N in Eq. (18). In the discussion below we consider the case of three RH neutrinos, $a, k = 1, 2, 3$.

After electroweak symmetry breaking, the neutrino Yukawa interactions in Eq. (18) generate a Dirac neutrino mass term $(m_D)_{\alpha b}\,\overline{\nu}_{L\alpha} N_b$, where $m_D = \lambda\, v/\sqrt{2}$ and $v = 246$ GeV is the vacuum expectation value of the SM Higgs. Under the assumption that $v \ll M_k$, the heavy RH neutrinos $\mathcal{N}_k$ provide at tree-level an effective Majorana mass term for the active neutrino fields $\nu_{L\alpha}$, i.e., $\frac{1}{2}(M_\nu)_{\alpha\beta}\,\overline{\nu}_{L\alpha}\nu^c_{L\beta}$, with

$$M_\nu \;\approx\; -\frac{1}{2}\, v^2\, \lambda\, M_N^{-1}\, \lambda^T \;=\; -m_D\, M_N^{-1}\, m_D^T \;=\; U_\nu\, M_\nu^{\rm diag}\, U_\nu^T\,, \tag{19}$$

$M_\nu^{\rm diag}$ containing the light neutrino masses m_i as diagonal entries. Taking indicatively $M_\nu \sim 0.05$ eV and $m_D \sim 200$ GeV, one predicts a seesaw lepton number violating scale: $M_N \sim M_k \sim 10^{14}$ GeV.

In the early Universe, these RH neutrinos are produced by scattering processes involving SM particles in thermal equilibrium at temperatures $T \gtrsim M_k$. Their out-of-equilibrium decays generate a lepton asymmetry which is partially converted into a non-zero baryon asymmetry by fast sphaleron processes [108], a mechanism called *thermal leptogenesis* [109]. A pedagogical discussion of this mechanism is given in [110].

The cosmological baryon asymmetry $Y_{\Delta B}$ can be expressed in terms of the ratio between the net baryon number density and the entropy density of the Universe

$$Y_{\Delta B} \;\equiv\; \left.\frac{n_B - n_{\bar B}}{s}\right|_0 \;=\; (8.65 \pm 0.09) \times 10^{-11}\,, \tag{20}$$

where the quoted bfv and $1\,\sigma$ error are taken from [49] and the subscript "0" refers to the present epoch.

If the spectrum of the heavy RH neutrinos $\mathcal{N}_k$ is hierarchical, with $M_1 \ll M_2 \lesssim M_3$, most of the baryon asymmetry is produced by the out-of-equilibrium decay of the lightest state, $\mathcal{N}_1$, at temperatures $T \lesssim M_1$, provided the corresponding neutrino Yukawa couplings have sufficiently large CP-violating phases. For $M_1 \gtrsim 10^{13}$ GeV one has within the SM (see, e.g., [111])

$$Y_{\Delta B} \approx -\,1.39 \times 10^{-3}\, \epsilon_1\, \eta\,, \tag{21}$$

where $10^{-3} \lesssim \eta \lesssim 1$ is an efficiency factor which takes into account the washout of the lepton asymmetry [112] by lepton number violating processes occurring in the thermal bath, whereas ϵ_1 denotes the total CP asymmetry in $\mathcal{N}_1$ decays.

For a RH neutrino $\mathcal{N}_k$ decaying into a (given) lepton (of flavor α), the (flavored) CP asymmetry $\epsilon_{k(\alpha)}$ is defined as

$$\begin{aligned}
\epsilon_{k\alpha} \equiv\; & \frac{\Gamma\left(\mathcal{N}_k \to \phi\, \ell_\alpha\right) \,-\, \Gamma\left(\mathcal{N}_k \to \bar\phi\, \bar\ell_\alpha\right)}{\sum_\beta \left[\Gamma\left(\mathcal{N}_k \to \phi\, \ell_\beta\right) \,+\, \Gamma\left(\mathcal{N}_k \to \bar\phi\, \bar\ell_\beta\right)\right]} \\
=\; & \frac{1}{4\pi v^2\, (\hat m_D^\dagger \hat m_D)_{kk}} \sum_{j\neq k} \Big\{ {\rm Im}\left((\hat m_D^\dagger \hat m_D)_{kj} (\hat m_D)^*_{\alpha k} (\hat m_D)_{\alpha j} \right)\, f(M_j/M_k) \\
& + {\rm Im}\left((\hat m_D^\dagger \hat m_D)_{jk} (\hat m_D)^*_{\alpha k} (\hat m_D)_{\alpha j} \right)\, g\left(M_j/M_k\right) \Big\}
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\epsilon_k \equiv\; & \sum_{\alpha=e,\mu,\tau} \epsilon_{k\alpha} \\
=\; & \frac{1}{4\pi v^2\, (\hat m_D^\dagger \hat m_D)_{kk}} \sum_{j\neq k} {\rm Im}\left((\hat m_D^\dagger \hat m_D)^2_{kj} \right)\, f(M_j/M_k)\,,
\end{aligned} \tag{23}$$

where $f(x)$ and $g(x)$ are known loop functions [113], namely

$$f(x) = x\left[\frac{1}{1-x^2} + 1 - (1+x^2)\ln\left(1 + \frac{1}{x^2}\right)\right] ,$$
$$g(x) = \frac{1}{1-x^2} . \tag{24}$$

In Eq. (22) $\hat{m}_D = m_D U_R$ is the Dirac neutrino mass matrix in the RH neutrino mass basis. Successful leptogenesis is possible for the lightest neutrino mass $m_0 \lesssim 0.1$ eV [111, 114, 115], which is compatible with the current constraints on m_0. In certain cases of IH neutrino mass spectrum and values of 5×10^{-4} eV $\lesssim m_3 \lesssim 10^{-2}$ eV one can have very strong amplification of the baryon asymmetry produced via leptogenesis with respect to the case of $m_3 = 0$ [116].

The expression of the baryon asymmetry given in Eq. (21) is modified in the case the RH neutrino mass spectrum is not strongly hierarchical and/or flavor dynamics becomes relevant in leptogenesis [117–120], which is the case if $M_1 \lesssim 10^{12}$ GeV (see [121] for a detailed discussion of flavor effects in leptogenesis). In general, within the *flavored leptogenesis* regime, the physical CP phases of the neutrino Yukawa couplings $\lambda_{\alpha b}$ are not directly related to the CP phases potentially measurable at low energies, i.e. the Dirac phase δ and the two Majorana phases α and β in Eq. (2) and Eq. (3) (see, e.g., [122–126]).

However, low energy CP phases may also play a crucial role in having successful leptogenesis in the case when flavor effects are relevant [116, 127–132]. In fact, one can show that if 10^8 GeV $\lesssim M_1 \lesssim 10^{12}$ GeV, CP violation necessary for successful leptogenesis can be provided entirely by the CP phases in the lepton mixing matrix. In particular, under the assumption that the only source of CP violation in the seesaw Lagrangian in Eq. (18) is given by the Dirac phase δ in the PMNS mixing matrix, the observed value of $Y_{\Delta B}$ can be achieved, provided $M_1 \lesssim 5 \times 10^{11}$ GeV and $\sin\theta_{13}|\sin\delta| \gtrsim 0.09$ ($|J_{\rm CP}| \gtrsim 0.02$) [127, 128]. The predicted lower bound on $\sin\theta_{13}$ ($|J_{\rm CP}|$) is compatible with current hints of maximal CP violation in neutrino oscillations, i.e. $\delta \approx 3\pi/2$. One can also show that even if additional sources of CP violation are introduced in the seesaw Lagrangian, there are large regions of the parameter space, where the observed baryon asymmetry is reproduced, only if we assume non-trivial low energy CP-violating Dirac and/or Majorana phases in the lepton mixing matrix [129, 130]. For example, in the case of 5×10^{10} GeV $\lesssim M_1 \lesssim 10^{12}$ GeV, IH light neutrino mass spectrum and $\sin\theta_{13}\cos\delta \lesssim -0.1$ a sufficiently large baryon asymmetry is possible, only if the requisite CP violation is provided by the non-trivial Majorana phase α in the lepton mixing matrix, independently of possible high energy CP phases in the seesaw Lagrangian.

A direct connection between low energy CP violation in the lepton sector and the source of CP violation required to have successful leptogenesis in the early Universe is possible in several seesaw scenarios which implement flavor (and CP) symmetries in order to explain the neutrino oscillation data. We will explore in the next section the implications for leptogenesis in some representative scenarios [133–137].

4. Flavor (and CP) symmetries for leptogenesis

In this section we discuss leptogenesis in scenarios with a flavor (and CP) symmetry that can constrain the mixing angles (and CP phases) in the high and low energy sector. We focus on neutrinos being Majorana particles and on the type I seesaw mechanism with three RH neutrinos as the source of light neutrino masses. The decay of RH neutrinos $\mathcal{N}_k$ leads to the generation of the CP asymmetries $\epsilon_{k(\alpha)}$ that are converted into the cosmological baryon asymmetry $Y_{\Delta B}$.

It is well-known that flavor (and CP) symmetries are a powerful tool for explaining the measured values of the mixing angles in the lepton sector and for making predictions for the yet-to-be-measured Dirac and Majorana phases. The two central assumptions [138–140] are that the three generations of leptons form a (irreducible) triplet, $\ell_\alpha \sim \mathbf{3}$, of a flavor symmetry G_f (and CP) and that such symmetry is not broken arbitrarily, but rather to non-trivial residual symmetries G_e in the charged lepton and G_ν in the neutrino sector, respectively.

We present examples of different scenarios: the baryon asymmetry $Y_{\Delta B}$ is either generated via unflavored [133–135, 137, 141–145] or via flavored leptogenesis [136, 143, 146, 147]; these scenarios possess either a flavor symmetry G_f only [133–135, 141–144], or G_f together with a CP symmetry [136, 137, 145, 146] that in general acts non-trivially on flavor space; and the generation of non-vanishing $Y_{\Delta B}$ might [135, 137, 142–144, 147] or might not [133, 134, 136, 141, 146] require corrections to the mentioned breaking scheme of flavor (and CP) symmetries. If it does, the smallness of these corrections can also explain the smallness of $Y_{\Delta B}$.

Moreover, we briefly highlight the impact of the constraints on Majorana phases on the effective Majorana mass $|m_{\beta\beta}|$, as discussed in [136, 137, 148–150].

4.1. *Impact of G_f (and CP) on lepton mixing parameters*

As shown in [151, 152], maximal atmospheric mixing and $\theta_{13} = 0$, the latter by now excluded experimentally to high degree, can be obtained from a light neutrino mass matrix M_ν that is invariant under the exchange of its second and third rows and columns in the charged lepton mass basis (M_e is diagonal). Thus, the presence of different residual flavor symmetries, i.e. the $\mu - \tau$ exchange symmetry $G_\nu = Z_2^{\mu\tau}$ in the neutrino sector, represented by the three-by-three matrix $P^{\mu\tau}$ that exchanges the second and third (lepton) generation, and the discrete charged lepton flavor symmetry $G_e = Z_3$ (or in its continuous version $G_e = U(1)$), leads to the prediction of two of the three lepton mixing angles. The maximal size of G_ν in the case of three generations of Majorana neutrinos is $G_\nu = Z_2 \times Z_2$, as shown in [153], and, if it is realized, also the solar mixing angle θ_{12} can be fixed. The most prominent example is tribimaximal (TB) lepton mixing with $\sin^2\theta_{12} = 1/3$, proposed in [154, 155]. This mixing pattern can be obtained from the flavor symmetries $G_f = A_4$, see [156–159], and $G_f = S_4$, see [160], and their peculiar breaking to $G_e = Z_3$ in the charged lepton

and to $G_\nu = Z_2 \times Z_2$ in the neutrino sector.[h] Much effort has been made in order to understand the observed values of the lepton mixing angles with the help of flavor symmetries G_f and their breaking patterns [161–164] (for reviews see [165–168]). In the most predictive version of this approach, in which all lepton generations are distinguished by the residual symmetries and the maximal symmetry $G_\nu = Z_2 \times Z_2$ is chosen, the charged lepton mass matrix M_e and the light neutrino mass matrix M_ν are constrained by

$$g_e(\mathbf{3})^\dagger \, M_e \, M_e^\dagger \, g_e(\mathbf{3}) = M_e \, M_e^\dagger \;\; \text{and} \;\; g_{\nu,i}(\mathbf{3})^\dagger \, M_\nu \, g_{\nu,i}(\mathbf{3})^* = M_\nu \; , \; i = 1, \, 2 \; , \qquad (25)$$

with $g_e(\mathbf{3})$ and $g_{\nu,i}(\mathbf{3})$ being the generators of G_e and G_ν, respectively, in the (irreducible) three-dimensional representation $\mathbf{3}$. In this case the numerical values of all three lepton mixing angles can be explained and the Dirac phase δ is predicted. The main result of surveys [161–164] of flavor symmetries G_f being subgroups of $SU(3)$ or $U(3)$ is the following: all mixing patterns, that are in accordance with the experimental data at the $3\,\sigma$ level or better, predict a lower bound on the solar mixing angle, $\sin^2\theta_{12} \gtrsim 1/3$, as well as CP-conserving values of δ. In less constrained approaches also values of δ different from 0 and π can be obtained [140, 150, 169–173].

In view of the recent experimental indications of δ close to $3\,\pi/2$ different ways to obtain lepton mixing patterns with such feature have been explored. One very promising approach involves in addition to G_f a CP symmetry [174] (see also [175, 176]). The latter acts in general non-trivially on flavor space [177] and can be represented in the three-dimensional representation $\mathbf{3}$, under the assumption that CP is an involution, by a CP transformation $X(\mathbf{3})$ which is a unitary and symmetric three-by-three matrix. A prominent example of this type of CP symmetry is the $\mu - \tau$ reflection symmetry [178] where $X(\mathbf{3}) = P^{\mu\tau}$. Again, we assume that non-trivial residual symmetries are preserved in the charged lepton and neutrino sector, respectively. In the charged lepton mass basis imposing $X(\mathbf{3}) = P^{\mu\tau}$ alone on the light neutrino mass matrix M_ν, i.e. $P^{\mu\tau} \, M_\nu^* \, P^{\mu\tau} = M_\nu$, is sufficient for achieving maximal atmospheric mixing, a maximal CP phase δ ($\delta = \pi/2$ or $\delta = 3\,\pi/2$) and CP-conserving Majorana phases, while the values of θ_{13} and θ_{12} are not fixed [178]. The number of free parameters in lepton mixing can be reduced to a single real one, if M_ν is additionally invariant under a residual flavor symmetry Z_2 [174]. Compared to the previous approach, G_e is chosen in the same way, i.e. as a subgroup of G_f, and the form of G_ν is changed from $G_\nu = Z_2 \times Z_2$ to the direct product of a Z_2 subgroup of G_f and the CP symmetry, $G_\nu = Z_2 \times CP$. Consequently, the constraints on M_ν read in general

$$g_\nu(\mathbf{3})^\dagger \, M_\nu \, g_\nu^*(\mathbf{3}) = M_\nu \;\; \text{and} \;\; X(\mathbf{3}) \, M_\nu^* \, X(\mathbf{3}) = M_\nu \; . \qquad (26)$$

One free real parameter θ is introduced in the lepton mixing pattern, since the residual flavor symmetry in the neutrino sector is reduced from $Z_2 \times Z_2$ to Z_2

[h] In the case of $G_f = A_4$, G_ν is partly contained in G_f and partly accidental, whereas G_ν is a subgroup of G_f in the case of $G_f = S_4$.

only. The predictive power regarding CP phases is, however, enhanced, as CP is involved as symmetry, and both Majorana phases are also predicted. Hence, all lepton mixing parameters only depend on θ, that has to be adjusted to a particular value in order to obtain lepton mixing angles in accordance with the experimental data. By now many flavor symmetries G_f have been combined with CP and several interesting mixing patterns have been discussed in [148, 179–183].

Corrections to this picture such as those arising from renormalization group running and from perturbations of the symmetry breaking pattern strongly depend on the explicit model, e.g. on the light neutrino mass spectrum [184, 185], whether the theory is supersymmetric or not [156–159], which auxiliary symmetries are present in the model. They are assumed to have a small impact on lepton mixing parameters.

Throughout the analyses focussing on lepton mixing patterns the mechanism that generates light neutrino masses is not specified [138–140, 174]. However, these approaches can be combined with various generation mechanisms for light neutrino masses. In particular, they can be implemented in scenarios with the type I seesaw mechanism. The RH neutrinos N_a in Eq. (18) are assigned to the same triplet **3** as the lepton generations ℓ_α. Two different mass matrices, the Dirac neutrino mass matrix m_D, arising from the Yukawa couplings of RH neutrinos to LH leptons, and the Majorana mass matrix M_N of the RH neutrinos, are present in the neutrino sector. Several possibilities can be considered that all can lead to the same results for lepton mixing: m_D is invariant under the entire flavor (and CP) symmetry, while M_N enjoys the residual symmetry G_ν; or M_N is invariant under the entire flavor (and CP) symmetry and m_D only under G_ν; or both mass matrices, m_D and M_N, are only constrained by G_ν. In the scenarios we discuss here, we focus on the first [135, 137] and the third possibility [134, 136]. The second possibility can also be very interesting, if resonant leptogenesis is the mechanism generating $Y_{\Delta B}$ [8].

4.2. *Predictions for leptogenesis*

We first make some general observations regarding the size of the CP asymmetries $\epsilon_{k(\alpha)}$ and their phase dependence. Then, we discuss examples for flavored [136] as well as unflavored leptogenesis [134, 135, 137] in scenarios with flavor symmetries G_f [134, 135] as well as in scenarios with flavor and CP symmetries [136, 137] and under the assumption of different residual symmetries G_e and G_ν. For concreteness, we consider non-supersymmetric theories. However, all results can be easily adapted to supersymmetric models. Moreover, we focus on scenarios in which RH neutrino masses are not degenerate and hence results may differ for resonant leptogenesis.

4.2.1. *General observations*

First, we discuss the case in which LH leptons and RH neutrinos form irreducible triplets **3** of a flavor group G_f (and there might or might not be a CP symmetry

present), since this is the most common situation for non-abelian G_f.[i] The Dirac neutrino mass matrix m_D can be expanded, like all other mass matrices, in terms of the (small, positive) flavor (and CP) symmetry breaking parameter κ

$$m_D = m_D^0 + \delta m_D + \mathcal{O}(\kappa^2\, v) \tag{27}$$

with $m_D^0 \sim \mathcal{O}(v)$ and $\delta m_D \sim \mathcal{O}(\kappa\, v)$. Typically κ is of order $10^{-3} \lesssim \kappa \lesssim 10^{-1}$.

The combination $\hat{m}_D^\dagger \hat{m}_D$ can be expanded as

$$\hat{m}_D^\dagger \hat{m}_D \approx U_R^\dagger \left((m_D^0)^\dagger m_D^0 + (m_D^0)^\dagger\, \delta m_D + (\delta m_D)^\dagger\, m_D^0 \right) U_R\ . \tag{28}$$

In the limit $\kappa \to 0$ this combination is diagonal in flavor space, since the first term on the right-hand side in Eq. (28) is proportional to the identity matrix. Hence, the CP asymmetries vanish for unflavored as well as flavored leptogenesis. Once corrections $\delta m_D \sim \mathcal{O}(\kappa\, v)$ are included, we find in general for unflavored leptogenesis

$$\epsilon_k \sim \sum_{j \neq k} \mathrm{Im} \left(\left(\hat{m}_D^\dagger \hat{m}_D \right)^2_{kj} \right) \propto \kappa^2 \tag{29}$$

and for flavored leptogenesis

$$\epsilon_{k\alpha} \sim \sum_{j \neq k} \mathrm{Im} \left(\left(\hat{m}_D^\dagger \hat{m}_D \right)_{\left[\begin{smallmatrix} kj \\ jk \end{smallmatrix}\right]} (\hat{m}_D)^*_{\alpha k} (\hat{m}_D)_{\alpha j} \right) \propto \kappa\, . \tag{30}$$

Realizations with specific flavor symmetries can be found in [137, 143]. Note that the combination $\hat{m}_D^\dagger \hat{m}_D$ does not depend, up to order κ, on the phases present in δm_D, if the matrix $(m_D^0)^\dagger\, \delta m_D$ is symmetric (for further options see [137]), since in this case the second and third term on the right-hand side in Eq. (28) are complex conjugated to each other.

If a Z_2 symmetry determines the relevant structure of m_D, either because it is a factor of the abelian flavor symmetry G_f or m_D^0 vanishes and the leading term is given by δm_D, invariant under Z_2 only, the CP asymmetries $\epsilon_{k(\alpha)}$ do not vanish and are in general not suppressed by a small parameter [134, 136].

If the theory possesses a CP symmetry that is left intact in the neutrino sector, it is true that the CP asymmetries ϵ_k vanish in the case of unflavored leptogenesis, but $\epsilon_{k\alpha}$ can be non-zero and thus flavored leptogenesis can explain the baryon asymmetry $Y_{\Delta B}$. A proof of this statement can straightforwardly be obtained by observing that the two conditions

$$X(\mathbf{3})^*\, m_D^\dagger m_D\, X(\mathbf{3}) = (m_D^\dagger m_D)^* \quad \text{and} \quad X(\mathbf{3})\, M_N\, X(\mathbf{3}) = M_N^*\ , \tag{31}$$

which express the invariance of m_D and M_N under the CP transformation $X(\mathbf{3})$, require the following form of $m_D^\dagger m_D$ and M_N

$$\begin{aligned} m_D^\dagger m_D &= \Omega\, O_D\, \mathrm{diag}(m_{d\,i}^2)\, (\Omega\, O_D)^\dagger\, , \\ M_N &= (\Omega\, O_R\, K_R)^*\, M_N^{\mathrm{diag}}\, (\Omega\, O_R\, K_R)^\dagger \end{aligned} \tag{32}$$

[i] For the reader interested in other cases we refer to [143].

where Ω is unitary and determined by $X(\mathbf{3})$, $X(\mathbf{3}) = \Omega\,\Omega^T$, $O_{D,R}$ are three-by-three rotation matrices, $m_{d\,i}^2 > 0$, $M_N^{\rm diag}$ contains the RH neutrino masses M_k as diagonal entries and K_R is a diagonal matrix with ± 1 and $\pm i$, accounting for the CP parities of the RH neutrinos. Consequently, the application of $U_R = \Omega\, O_R\, K_R$ to $m_D^\dagger m_D$ shows that the off-diagonal elements of $\hat{m}_D^\dagger \hat{m}_D$ can at most carry a factor $\pm i$ arising from K_R. Similar observations are also reported in [146].

We exemplify these different situations in the following with well-known examples taken from the literature [134–137].

4.2.2. *Leptogenesis in scenarios with flavor symmetries*

We present examples which show the predictive power of different types of flavor and residual symmetries with regard to the CP asymmetries ϵ_k.

Case of residual symmetry Z_2 in neutrino sector. We consider an example with $\mu-\tau$ exchange symmetry among neutrinos [151, 152] and charged leptons invariant under a discrete or continuous charged lepton flavor symmetry. In [133] the results for unflavored leptogenesis have been studied for a strongly hierarchical RH neutrino mass spectrum and with two or three RH neutrinos. It has been shown that for two RH neutrinos the CP asymmetry ϵ_1 vanishes in the limit of exact $\mu-\tau$ exchange symmetry, while for three RH neutrinos a non-zero CP asymmetry is obtained. The latter result has also been found in [134, 141]. For $M_1 \ll M_{2,3}$ it is known that the CP asymmetry ϵ_1, arising from the decay of the lightest RH neutrino $\mathcal{N}_1$, can be written as

$$\epsilon_1 \approx -\frac{3\,M_1}{8\pi v^2}\frac{{\rm Im}\Big(\left(\hat{m}_D^T M_\nu^* \hat{m}_D\right)_{11}\Big)}{(\hat{m}_D^\dagger \hat{m}_D)_{11}} = \frac{3\,M_1}{8\pi v^2}\frac{{\rm Im}\left(R_{12}^2\,\Delta m_{21}^2 + R_{13}^2\,\Delta m_{31}^2\right)}{\left(\sum_j |R_{1j}|^2\, m_j\right)}\,. \tag{33}$$

Here we have approximated the loop function $f(x)$ in Eq. (24) for strongly hierarchical RH neutrino masses. The R matrix of the Casas-Ibarra parametrization [186] is defined as

$$R = i\,(M_N^{\rm diag})^{-1/2}\, m_D^T\, U_\nu^*\, (M_\nu^{\rm diag})^{-1/2} \tag{34}$$

in the charged lepton mass basis. If the neutrino sector is invariant under $\mu-\tau$ exchange symmetry, i.e. both, m_D and M_N, are invariant under the application of $P^{\mu\tau}$, the R matrix has block structure with $R_{13} = R_{23} = R_{31} = R_{32} = 0$. Then, ϵ_1 is proportional to the solar mass squared difference only [134]

$$\epsilon_1 \approx \frac{3\,M_1}{8\pi v^2}\frac{{\rm Im}\left(\sin^2 z\right)\,\Delta m_{21}^2}{\left(|\cos z|^2\; m_1 + |\sin z|^2\; m_2\right)} \tag{35}$$

with z being the complex angle parametrizing the R matrix. Since vanishing reactor mixing angle θ_{13} is experimentally excluded to high degree, $\mu-\tau$ exchange symmetry cannot be exact in the charged lepton mass basis. An example in which $\mu-\tau$

exchange symmetry is broken in the RH neutrino sector is provided in [134]. The element R_{13} of the R matrix then becomes proportional to θ_{13}, $R_{13} \propto \theta_{13}$, and thus the CP asymmetry ϵ_1 can be cast into the form

$$\epsilon_1 \propto \left(c_1\, \Delta m_{21}^2 + c_2\, \Delta m_{31}^2\, \theta_{13}^2\right) \,, \tag{36}$$

with $c_{1,2}$ depending on, e.g., the elements R_{ij} with $i, j = 1, 2$ of the R matrix.

For figures of $Y_{\Delta B}$ in different scenarios with $\mu - \tau$ exchange symmetry in the neutrino sector see [134, 141].

Case of residual symmetry $Z_2 \times Z_2$ in neutrino sector. We discuss results for a model with $G_f = A_4$ that leads to TB lepton mixing [158]. This model is originally formulated in a supersymmetric context. However, this does not have an impact on the relation of the CP asymmetries ϵ_k to the low energy CP phases. The residual symmetry $G_e = Z_3$ constrains charged leptons to have a diagonal mass matrix, while the Dirac neutrino mass matrix m_D is invariant under the entire flavor group and the RH neutrino mass matrix M_N has $G_\nu = Z_2 \times Z_2$ as symmetry. The actual form of m_D and M_N at leading order is

$$m_D = m_D^0 = \overline{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad M_N = \begin{pmatrix} A + 2\,B & -B & -B \\ -B & 2\,B & A - B \\ -B & A - B & 2\,B \end{pmatrix} \tag{37}$$

with A and B being complex mass parameters, while $\overline{m} > 0$ can be achieved via a field redefinition. The light neutrino mass matrix arising from the type I seesaw mechanism leads to TB lepton mixing. Light and heavy neutrino masses m_i and M_k are strongly correlated, since the form of m_D is trivial in flavor space, i.e.

$$m_i = \frac{\overline{m}^2}{M_i} \,. \tag{38}$$

In addition, they only depend on two complex parameters. They, hence, fulfill the following light neutrino mass sum rule involving the Majorana phases α and β

$$\frac{1}{m_1} - \frac{e^{2\,i\,\beta}}{m_3} = \frac{2\,e^{2\,i\,\alpha}}{m_2} \,. \tag{39}$$

Consequently, the mass spectra of light and heavy neutrinos are non-hierarchical and can have both orderings. In turn, all three RH neutrinos contribute to the generation of $Y_{\Delta B}$. The off-diagonal elements of $\hat{m}_D^\dagger \hat{m}_D$ vanish at leading order and hence also the CP asymmetries $\epsilon_{k(\alpha)}$.

In explicit models, however, the form of m_D and M_N as well as M_e receives corrections at the subleading level which break the residual symmetries G_ν and G_e, respectively. In the particular A_4 model the dominant correction δm_D turns out to be invariant under G_e

$$\delta m_D = \overline{m} \begin{pmatrix} 2\,y_1 & 0 & 0 \\ 0 & 0 & -y_1 - y_2 \\ 0 & -y_1 + y_2 & 0 \end{pmatrix} \kappa \;\; \text{with} \;\; y_{1,2} \;\; \text{complex.} \tag{40}$$

Including δm_D when computing ϵ_k in the case of unflavored leptogenesis, we find [135]

$$\epsilon_1 \propto \kappa^2 \left(6\,\mathrm{Re}\,(y_1)^2\; f\left(\frac{m_1}{m_2}\right) \sin 2\,\alpha + \mathrm{Re}\,(y_2)^2\; f\left(\frac{m_1}{m_3}\right) \sin 2\,\beta \right) \tag{41}$$

and similar results for ϵ_2 and ϵ_3. This shows that the CP asymmetries crucially depend on the two Majorana phases α and β and, since the matrix $(m_D^0)^\dagger\, \delta m_D$ is symmetric, the imaginary parts of the parameters $y_{1,2}$ of the correction do not enter at $\mathcal{O}(\kappa^2)$. $f(x)$ is the loop function in Eq. (24) and its argument can be written in terms of light neutrino masses with the help of Eq. (38). The corrections to the mass matrices affect in general also the predictions for lepton mixing. However, since κ is small, these corrections only have a minor impact [158].

In order to compute $Y_{\Delta B}$ the efficiency factors η_{ij} have to be taken into account [135]. In this type of model RH neutrinos couple at leading order to orthogonal linear combinations of lepton flavors and thus the contributions $Y_{\Delta B,k}$ from the decay of the three different RH neutrinos $\mathcal{N}_k$ to $Y_{\Delta B}$ can be summed incoherently. For figures of $Y_{\Delta B}$ see [135].

4.2.3. *Leptogenesis in scenarios with flavor and CP symmetries*

As has been argued, imposing a CP symmetry in addition to a flavor symmetry increases the predictive power of the approaches regarding CP phases so that it can become possible to correlate the sign of $Y_{\Delta B}$ with the low energy CP phases. We exemplify this in the following with the help of two concrete scenarios taken from [136, 137].

Case of residual CP symmetry in neutrino sector. We discuss a setup [136] in which the charged lepton mass matrix is diagonal due to the choice of G_e and M_ν is constrained by $\mu-\tau$ reflection symmetry [178], $X(\mathbf{3}) = P^{\mu\tau}$. In particular, the Majorana mass matrix M_N of the RH neutrinos and the Dirac neutrino mass matrix m_D, both are invariant under $\mu-\tau$ reflection symmetry. As already shown above for a general CP symmetry, preserved in the neutrino sector, the CP asymmetries ϵ_k vanish in the case of unflavored leptogenesis. This observation has also been made in [145]. In contrast, $\epsilon_{k\alpha}$ are normally non-zero. Thus, we consider in the following RH neutrinos with masses below 10^{12} GeV so that flavor effects become relevant when computing the CP asymmetries.

We note that from

$$X(\mathbf{3})^*\, m_D\, X(\mathbf{3}) = P^{\mu\tau}\, m_D\, P^{\mu\tau} = m_D^* \tag{42}$$

together with Eq. (32) the following relation can be derived for $\hat{m}_D$

$$\hat{m}_D^* = P^{\mu\tau}\, \hat{m}_D\, K_R^2\,, \tag{43}$$

K_R^2 being a diagonal matrix with entries ± 1.

Assuming that RH neutrinos are strongly hierarchical, $M_1 \ll M_{2,3}$, we only consider the CP asymmetries $\epsilon_{1\alpha}$, generated in the decay of the RH neutrino $\mathcal{N}_1$. In order to evaluate the expression in Eq. (22) for $\epsilon_{1\alpha}$ we use Eq. (43), taking into account all possible choices for K_R^2. We find that

$$\epsilon_{1e} = 0\,, \tag{44}$$

whereas

$$\epsilon_{1\mu} = -\epsilon_{1\tau} \neq 0 \tag{45}$$

in general. This leads to vanishing ϵ_1, as expected. Since $\epsilon_{1\mu}$ and $\epsilon_{1\tau}$ have opposite sign, but equal magnitude, $Y_{\Delta B}$ can only be explained, if we consider the regime in which the τ flavor alone can be distinguished by its fast Yukawa interaction in the primordial plasma. This leads to 10^9 GeV as lower bound on the RH neutrino masses.

The source of CP violation that leads to non-vanishing CP asymmetries $\epsilon_{1\mu}$ and $\epsilon_{1\tau}$ can be traced back to the maximal Dirac phase δ. In order to see this consider

$$\begin{aligned} \epsilon_{1\alpha} &\approx -\frac{3\,M_1}{8\pi v^2}\frac{\mathrm{Im}\Big(\left(\hat{m}_D^T M_\nu^*\right)_{1\alpha}(\hat{m}_D)_{\alpha 1}\Big)}{(\hat{m}_D^\dagger \hat{m}_D)_{11}} \\ &= \frac{3\,M_1}{8\pi v^2}\frac{\sum_{i,j} m_i\sqrt{m_i\,m_j}\,\mathrm{Im}\left(R_{1i}\,R_{1j}\,U^*_{\nu,\alpha i}\,U_{\nu,\alpha j}\right)}{\left(\sum_k |R_{1k}|^2\,m_k\right)}\,. \end{aligned} \tag{46}$$

Furthermore, we note that the R matrix fulfills

$$R^* = -K_R^2\,R\,K_\nu^2\,, \tag{47}$$

with K_ν encoding the CP parities of the light neutrinos. Thus, K_ν^2 is a diagonal matrix with entries ± 1. This equation can be derived using Eq. (43) together with the constrained form of U_ν. Given Eq. (47) the elements of the R matrix can only be real or imaginary, i.e. R can be written as

$$R = i\,K_R^*\,R^{(0)}\,K_\nu\,, \tag{48}$$

with $R^{(0)}$ being a real matrix. Plugging this information in Eq. (46), one can see that $\epsilon_{1\tau}$ (and hence also $\epsilon_{1\mu}$) is proportional to the Jarlskog invariant J_{CP} [63] and is consequently sourced from the low energy CP phase δ, that is maximal.

In Fig. 2 results for the magnitude of $Y_{\Delta B}$, normalized to the experimentally observed value, versus the element R_{12} of the R matrix are shown for neutrinos with NH and IH. We assume that $M_3 \gg M_{1,2}$ so that the RH neutrino $\mathcal{N}_3$ decouples and in turn the R matrix has block structure. The RH neutrino mass M_1 is set to its maximal admitted value $M_1 = 10^{12}$ GeV. For a smaller M_1 mass the magnitude of $Y_{\Delta B}$ has to be appropriately rescaled. Results for the two different values of δ, $\delta = \pi/2$ and $\delta = 3\,\pi/2$, and $Y_{\Delta B}$ positive are displayed in different line style. Different choices of K_R and K_ν are considered for the different curves and are indicated with (ij). As one can see, not all these choices lead to a large enough value of the magnitude of $Y_{\Delta B}$. For further details and results for other combinations (ij) see [136].

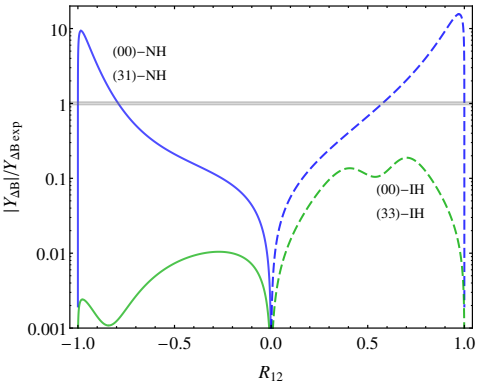

Fig. 2. Prediction of magnitude of $Y_{\Delta B}$, normalized to experimental observation (indicated by the horizontal gray lines), versus the element R_{12} of the R matrix in a scenario with $\mu - \tau$ reflection symmetry in the neutrino sector and a diagonal charged lepton mass matrix. Light neutrinos follow NH and IH for blue and green curves. Solid (dashed) lines refer to cases with the Dirac phase $\delta = 3\,\pi/2$ ($\delta = \pi/2$) and $Y_{\Delta B} > 0$. (ij) indicate different choices of K_R and K_ν. Figure adapted from [136].

Case of residual symmetry $Z_2 \times CP$ in neutrino sector. We present a scenario [137] in which a flavor and a CP symmetry determine lepton mixing and the sign of $Y_{\Delta B}$. We choose in the following as G_f one of the groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$. For details about these groups see [187, 188]. The CP symmetry depends in general on one or two parameters. The residual symmetries are $G_e = Z_3$ and $G_\nu = Z_2 \times CP$. The basis of G_f is chosen in such a way that M_e is diagonal. Like in the discussed A_4 model [158], the Dirac neutrino mass matrix m_D respects all symmetries imposed on the theory, while the Majorana mass matrix M_N of the RH neutrinos is only invariant under G_ν

$$m_D = m_D^0 = \overline{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad U_R^T\, M_N\, U_R = M_N^{\mathrm{diag}}\,, \tag{49}$$

with $U_R = \Omega\, R_{ij}(\theta)\, K_R$ and $\overline{m}$ positive. The form of Ω is determined by G_ν. The rotation $R_{ij}(\theta)$ in the (ij)-plane depends on θ, $0 \leq \theta < \pi$, that is the only free parameter in U_R. The RH neutrino masses M_k are also unconstrained. Applying the type I seesaw mechanism, we see that the lepton mixing matrix is given by U_R and, thus, θ is fixed by the constraint to accommodate the measured lepton mixing angles. The light neutrino masses m_i are, like in the A_4 model, inversely proportional to M_i, see Eq. (38). As expected, the CP asymmetries $\epsilon_{k(\alpha)}$ vanish at this level, because $(\hat{m}_D^\dagger \hat{m}_D)_{ij} = 0$ for $i \neq j$, and corrections are needed for $\epsilon_{k(\alpha)} \neq 0$. Similar to the A_4 model, we consider the dominant correction δm_D to be invariant under G_e, the residual symmetry in the charged lepton sector. The form of δm_D is then

$$\delta m_D = \overline{m} \begin{pmatrix} \frac{2}{\sqrt{3}}\, z_1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{3}}\, z_1 - z_2 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}}\, z_1 + z_2 \end{pmatrix} \kappa \quad \text{with} \quad z_{1,2} \quad \text{complex.} \tag{50}$$

It is convenient to parametrize the real parts of $z_{1,2}$ as $\mathrm{Re}(z_1) = z\, \cos\zeta$ and $\mathrm{Re}(z_2) = z\, \sin\zeta$, with $z > 0$ and $0 \leq \zeta < 2\,\pi$. In [148, 179] it has been shown that four

Table 1. Prediction of CP phases α, β' and δ from $\Delta(6 \cdot 8^2)$ and $CP(s)$, $G_e = Z_3$, $G_\nu = Z_2(4) \times CP(s)$. K_R is trivial [137, 179].

s	$\sin 2\,\alpha = \sin 2\,\beta'$	$\sin\delta$
1	$-1/\sqrt{2}$	± 0.936
2	1	-0.739
4	0	∓ 1

different types of mixing patterns, i.e. U_R, exist and we focus in the following on Case 1) and Case 3 b.1) of [179].

Case 1) predicts only the Majorana phase α to be non-trivial

$$|\sin 2\,\alpha| = \left|\sin\left(\frac{6\,\pi\,s}{n}\right)\right|, \quad \sin 2\,\beta' = 0\,, \quad \sin\delta = 0\,. \tag{51}$$

The integer parameter s characterizes the CP symmetry $CP(s)$ and lies in the interval $0 \leq s \leq n-1$. The CP asymmetries ϵ_k have a very simple form and we find [137]

$$\epsilon_1 \propto \kappa^2\,\cos^2(\theta+\zeta)\;f\left(\frac{m_1}{m_2}\right)\sin\left(\frac{6\,\pi\,s}{n}\right) \propto \kappa^2\,f\left(\frac{m_1}{m_2}\right) I_1\,(\theta \to \theta+\zeta) \tag{52}$$

and similar expressions for ϵ_2 and ϵ_3. This formula shows that the sign of ϵ_1 only depends on s, and hence α, and the light neutrino mass spectrum through the sign of the loop function $f(x)$. Furthermore, we recognize that the terms in ϵ_k have the same structure as the Majorana CP invariants I_i, $i = 1, 2, 3$ (for the definition of I_i see [189, 190]). As shown in [137], this not only holds for lepton mixing patterns derived from G_f and CP, but can also be found if a generic form of the lepton mixing matrix is considered.

Case 3 b.1) has a much richer phenomenology, since all CP phases take in general CP-violating values. We focus on the flavor group $\Delta(6 \cdot 8^2)$ with the index $n = 8$. The residual symmetry G_ν is characterized by two integer parameters m and s, $0 \leq m,\, s \leq n-1$. The symmetry $Z_2(m)$ is fixed by choosing $m = n/2 = 4$ such that the solar mixing angle is accommodated well. As a consequence, the sines of the Majorana phases coincide in size and also in sign, if K_R is trivial, and they depend only on the chosen CP symmetry $CP(s)$

$$|\sin 2\,\alpha| = |\sin 2\,\beta'| = \left|\sin\left(\frac{6\,\pi\,s}{n}\right)\right|. \tag{53}$$

Requiring that all three lepton mixing angles agree with the experimental data at the $3\,\sigma$ level or better, a lower bound on the size of the sine of the Dirac phase δ can be derived

$$|\sin\delta| \gtrsim 0.71\,. \tag{54}$$

For the admitted values of s, $s = 1$, $s = 2$ and $s = 4$, the CP phases are predicted as in Table 1. Since for $s = 1$ and $s = 4$ both, $\sin^2\theta_{23}$ and $1 - \sin^2\theta_{23}$, lie in

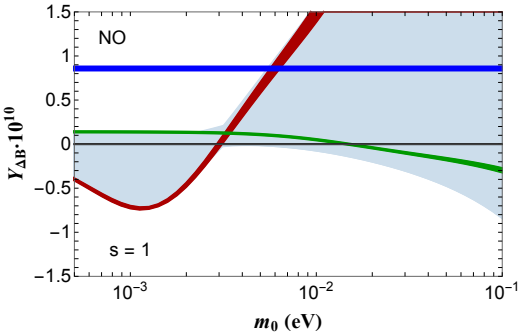

Fig. 3. Prediction of $Y_{\Delta B}$ versus the lightest neutrino mass m_0 from $\Delta(6{\cdot}8^2)$ and $CP(1)$, $G_e = Z_3$, $G_\nu = Z_2(4) \times CP(1)$. K_R is trivial. Light neutrino masses follow NO. Different colors refer to different ζ. The horizontal blue band indicates the experimentally preferred $3\,\sigma$ range of $Y_{\Delta B}$. Figure adapted from [137].

the experimentally preferred $3\,\sigma$ range for $\sin^2\theta_{23}$, $\sin\delta$ can be either positive or negative.

The prediction of $Y_{\Delta B}$ versus the lightest neutrino mass m_0 for the choice $s = 1$ is shown in Fig. 3. One sees that for small m_0, $m_0 \lesssim 3 \times 10^{-3}$ eV, negative $Y_{\Delta B}$ is preferred, while larger values of m_0 strongly favor positive $Y_{\Delta B}$. For the choice $s = 2$, predicting a different sign for $\sin 2\,\alpha$ and $\sin 2\,\beta'$ than $s = 1$, see Table 1, the sign of $Y_{\Delta B}$ changes accordingly, i.e. for $CP(2)$ positive $Y_{\Delta B}$ is achieved for small m_0, $m_0 \lesssim 4 \times 10^{-3}$ eV. For the choice $CP(4)$ in which both Majorana phases are CP-conserving, while the Dirac phase δ is maximal,[j] the sign of $Y_{\Delta B}$ cannot be predicted, since it is proportional to $\sin 2\,\zeta$ at leading order in κ, i.e. it depends on the relative sign of the real parts of the parameters in δm_D.

Flavored leptogenesis can also be considered in a scenario in which residual flavor and CP symmetries constrain the neutrino sector, as mentioned in [137] and studied in more detail in [146].

4.3. *Predictions for neutrinoless double beta decay*

One combination of the Majorana phases can be tested in $0\nu\beta\beta$ decay. The effective Majorana mass $|m_{\beta\beta}|$ depends on the CP phases α and $\beta^{(\prime)}$, the light neutrino masses, and the solar and reactor mixing angle, see Eq. (16).

In the example of $\mu-\tau$ reflection symmetry in the neutrino sector and a diagonal charged lepton mass matrix both Majorana phases are CP-conserving. In such a situation both, minimal and maximal, attainable values of $|m_{\beta\beta}|$ for a given lightest neutrino mass m_0 can be obtained, independent of the neutrino mass ordering. These results are shown in Fig. 1 (darker bands) and can also be found in [136]. Note that $0\nu\beta\beta$ decay experiments cannot distinguish between the CP-conserving scenario and the predictions of $\mu - \tau$ reflection symmetry, and information on the Dirac phase δ is needed, since in the first case δ is CP-conserving, while in the second one it is maximal.

The approach with a residual flavor and a CP symmetry in the neutrino sector is

[j]These results for the CP phases are also obtained in scenarios with $\mu-\tau$ reflection symmetry, see Sec. 4.1.

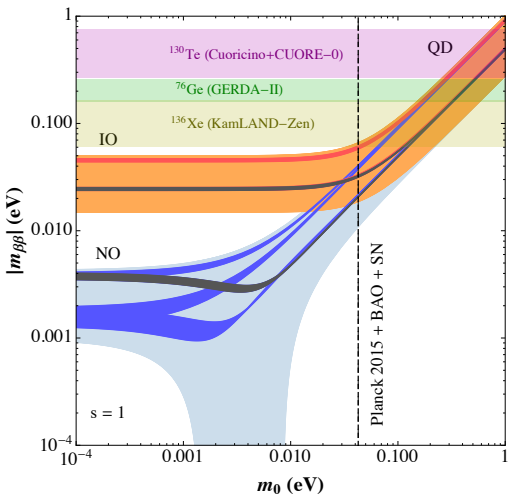

Fig. 4. Prediction of $|m_{\beta\beta}|$ versus m_0 from $\Delta(6 \cdot 8^2)$ and $CP(1)$, $G_e = Z_3$, $G_\nu = Z_2(4) \times CP(1)$. The orange (blue) areas refer to IO (NO) light neutrino masses. This approach strongly restricts the allowed values of $|m_{\beta\beta}|$ (darker colors) compared to the experimental constraints on the lepton mixing parameters alone (lighter colors). The choice K_R trivial is highlighted in dark grey. Exclusion limits from searches for $0\nu\beta\beta$ decay and from cosmology are also shown. Figure adapted from [137].

particularly powerful in constraining $|m_{\beta\beta}|$, since all CP phases and lepton mixing angles are highly correlated and predicted to lie in a small range, once the free parameter θ is fixed. We use the same flavor and CP symmetry as in the example for unflavored leptogenesis, see Fig. 3. The numerical values of the CP phases are shown in Table 1 and in Fig. 4 we display the prediction of $|m_{\beta\beta}|$ versus m_0 for neutrino masses with NO and IO for the choice $s = 1$ [137]. The predictive power of this approach is underlined by comparing the very limited ranges of $|m_{\beta\beta}|$ in this case (darker colors in Fig. 4) with the ranges of $|m_{\beta\beta}|$ allowed by the experimental constraints on the lepton mixing parameters alone (lighter colors in Fig. 4). In particular, we see that for NO $|m_{\beta\beta}|$ has a non-trivial lower bound and for IO the minimum attainable value of $|m_{\beta\beta}|$ is larger than 0.023 eV. For certain choices of K_R we find, indeed, that $|m_{\beta\beta}|$ is close to its upper limit.

Other examples of predictions of the Dirac and Majorana phases as well as of $|m_{\beta\beta}|$ in theories with combined flavor and CP symmetries can be found, e.g., in [149, 181].

5. Summary and conclusions

In this chapter we have reviewed the experimental status of neutrino data and future perspectives of measuring CP violation in the lepton sector. We have pointed out how a possible discovery of CP-violating effects in neutrino oscillations and/or the observation of $0\nu\beta\beta$ decay may reveal a deep connection between the origin of neutrino masses and lepton mixing and the production of the cosmological baryon asymmetry $Y_{\Delta B}$ via the leptogenesis mechanism.

We have presented different approaches in order to explain the observed lepton mixing angles with the help of flavor (and CP) symmetries and have shown which predictions for the yet-to-be-measured CP phases δ, α, $\beta^{(\prime)}$ can be made. We have

then discussed how such approaches can be implemented in scenarios in which light neutrino masses arise from the type I seesaw mechanism. In these scenarios $Y_{\Delta B}$ can be generated via the decay of heavy RH neutrinos. We have shown results for different variants and have elucidated in which ones and how the sign of $Y_{\Delta B}$ can be related to predictions for the CP phases δ, α, $\beta^{(\prime)}$.

We have focussed on the type I seesaw mechanism in this chapter. It is, however, also possible to implement the approaches with flavor (and CP) symmetries in scenarios with other generation mechanisms for neutrino masses. In case such scenarios offer a way to generate $Y_{\Delta B}$, flavor (and CP) symmetries can leave an imprint on the results for $Y_{\Delta B}$ as well. An analysis of leptogenesis in scenarios with type II [191–193] and type III [194] seesaw mechanisms and the flavor symmetry S_4 can be found in [195].

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References

[1] B. Pontecorvo, Mesonium and anti-mesonium, *Sov. Phys. JETP.* **6**, 429, (1957). [Zh. Eksp. Teor. Fiz. 33, 549 (1957)].

[2] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, *Prog. Theor. Phys.* **28**, 870–880, (1962).

[3] B. Pontecorvo, Neutrino Experiments and the Problem of Conservation of Leptonic Charge, *Sov. Phys. JETP.* **26**, 984–988, (1968). [Zh. Eksp. Teor. Fiz. 53,1717 (1967)].

[4] K. Nakamura and S. Petcov, Review of Particle Physics: Neutrino mass, mixing, and oscillations, *Chin. Phys.* **C40**(10), 100001, (2016).

[5] T. Kajita. Nobel Diploma. `http://www.nobelprize.org/nobel_prizes/physics/laureates/2015/kajita-diploma.html`.

[6] A. B. McDonald. Nobel Diploma. `http://www.nobelprize.org/nobel_prizes/physics/laureates/2015/mcdonald-diploma.html`.

[7] M. Drewes, B. Garbrecht, P. Hernández, M. Kekic, J. López-Pavón, J. Racker, N. Rius, J. Salvado, and D. Teresi, ARS Leptogenesis, `arXiv:1711.02862`, (2017).

[8] P. S. B. Dev, M. Garny, J. Klaric, P. Millington, and D. Teresi, Resonant enhancement in leptogenesis, `arXiv:1711.02863`, (2017).

[9] E. J. Chun, G. Cvetič, P. S. B. Dev, M. Drewes, C. S. Fong, B. Garbrecht, T. Hambye, J. Harz, P. Hernández, C. S. Kim, E. Molinaro, E. Nardi, J. Racker, N. Rius, and J. Zamora-Saa, Probing Leptogenesis, `arXiv:1711.02865`, (2017).

[10] J. Kopp, P. A. N. Machado, M. Maltoni, and T. Schwetz, Sterile Neutrino Oscillations: The Global Picture, *JHEP.* **05**, 050, (2013).

[11] S. Gariazzo, C. Giunti, M. Laveder, Y. F. Li, and E. M. Zavanin, Light sterile neutrinos, *J. Phys.* **G43**, 033001, (2016).

[12] G. H. Collin, C. A. Argüelles, J. M. Conrad, and M. H. Shaevitz, Sterile Neutrino Fits to Short Baseline Data, *Nucl. Phys.* **B908**, 354–365, (2016).

[13] F. Capozzi, C. Giunti, M. Laveder, and A. Palazzo, Joint short- and long-baseline constraints on light sterile neutrinos, *Phys. Rev.* **D95**(3), 033006, (2017).

[14] S. Gariazzo, C. Giunti, M. Laveder, and Y. F. Li, Updated Global 3+1 Analysis of Short-BaseLine Neutrino Oscillations, *JHEP.* **06**, 135, (2017).

[15] S. M. Bilenky, J. Hosek, and S. T. Petcov, On Oscillations of Neutrinos with Dirac and Majorana Masses, *Phys. Lett.* **B94**, 495–498, (1980).

[16] S. M. Bilenky and S. T. Petcov, Massive Neutrinos and Neutrino Oscillations, *Rev. Mod. Phys.* **59**, 671, (1987). [Erratum: Rev. Mod. Phys.60,575(1988)].

[17] S. T. Petcov, The Nature of Massive Neutrinos, *Adv. High Energy Phys.* **2013**, 852987, (2013).

[18] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, Global constraints on absolute neutrino masses and their ordering, *Phys. Rev.* **D95** (9), 096014, (2017).

[19] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, *JHEP.* **01**, 087, (2017).

[20] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola, and J. W. F. Valle, Status of neutrino oscillations 2017, `arXiv:1708.01186`, (2017).

[21] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Status of three-neutrino oscillation parameters, circa 2013, *Phys. Rev.* **D89**, 093018, (2014).

[22] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, Updated fit to three neutrino mixing: status of leptonic CP violation, *JHEP.* **11**, 052, (2014).

[23] K. Abe et al., Observation of Electron Neutrino Appearance in a Muon Neutrino Beam, *Phys. Rev. Lett.* **112**, 061802, (2014).

[24] H. A. Tanaka. T2K Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.

[25] F. P. An et al., Spectral measurement of electron antineutrino oscillation amplitude and frequency at Daya Bay, *Phys. Rev. Lett.* **112**, 061801, (2014).

[26] P. Adamson et al., First measurement of electron neutrino appearance in NOνA, *Phys. Rev. Lett.* **116**(15), 151806, (2016).

[27] P. Vahle. NOνA Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.

[28] F. P. An et al., Observation of electron-antineutrino disappearance at Daya Bay, *Phys. Rev. Lett.* **108**, 171803, (2012).

[29] F. P. An et al., Improved Measurement of Electron Antineutrino Disappearance at Daya Bay, *Chin. Phys.* **C37**, 011001, (2013).

[30] F. P. An et al., New Measurement of Antineutrino Oscillation with the Full Detector

Configuration at Daya Bay, *Phys. Rev. Lett.* **115**(11), 111802, (2015).
[31] Z. Yu. Daya Bay Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.
[32] J. K. Ahn et al., Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment, *Phys. Rev. Lett.* **108**, 191802, (2012).
[33] W. Choi. RENO Collaboration. Talk given at the WIN2015 International Workshop, MPIK Heidelberg, Germany, 8-13 June, 2015.
[34] K. Joo. RENO Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.
[35] Y. Abe et al., Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment, *Phys. Rev. Lett.* **108**, 131801, (2012).
[36] Y. Abe et al., First Measurement of θ_{13} from Delayed Neutron Capture on Hydrogen in the Double Chooz Experiment, *Phys. Lett.* **B723**, 66–70, (2013).
[37] Y. Abe et al., Improved measurements of the neutrino mixing angle θ_{13} with the Double Chooz detector, *JHEP.* **10**, 086, (2014). [Erratum: JHEP02,074(2015)].
[38] X. Qian and P. Vogel, Neutrino Mass Hierarchy, *Prog. Part. Nucl. Phys.* **83**, 1–30, (2015).
[39] S. T. Petcov and M. Piai, The LMA MSW solution of the solar neutrino problem, inverted neutrino mass hierarchy and reactor neutrino experiments, *Phys. Lett.* **B533**, 94–106, (2002).
[40] S. Pascoli and S. T. Petcov, The SNO solar neutrino data, neutrinoless double beta decay and neutrino mass spectrum, *Phys. Lett.* **B544**, 239–250, (2002).
[41] S. T. Petcov and T. Schwetz, Determining the neutrino mass hierarchy with atmospheric neutrinos, *Nucl. Phys.* **B740**, 1–22, (2006).
[42] F. Perrin, *Comptes Rendus.* **197**, 868, (1933).
[43] E. Fermi, Trends to a Theory of Beta Radiation. (In Italian), *Nuovo Cim.* **11**, 1–19, (1934).
[44] C. Kraus et al., Final results from phase II of the Mainz neutrino mass search in tritium beta decay, *Eur. Phys. J.* **C40**, 447–468, (2005).
[45] V. M. Lobashev, The search for the neutrino mass by direct method in the tritium beta-decay and perspectives of study it in the project KATRIN, *Nucl. Phys.* **A719**, 153–160, (2003).
[46] V. N. Aseev et al., An upper limit on electron antineutrino mass from Troitsk experiment, *Phys. Rev.* **D84**, 112003, (2011).
[47] K. N. Abazajian et al., Cosmological and Astrophysical Neutrino Mass Measurements, *Astropart. Phys.* **35**, 177–184, (2011).
[48] K. Eitel, Direct neutrino mass experiments, *Nucl. Phys. Proc. Suppl.* **143**, 197–204, (2005).
[49] P. A. R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, *Astron. Astrophys.* **594**, A13, (2016).
[50] S. Dell'Oro, S. Marcocci, M. Viel, and F. Vissani, Neutrinoless double beta decay: 2015 review, *Adv. High Energy Phys.* **2016**, 2162659, (2016).
[51] R. Acciarri et al., Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE), `arXiv:1512.06148`, (2015). FERMILAB-DESIGN-2016-02.
[52] R. Acciarri et al., Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE), `arXiv:1601.05471`, (2016). FERMILAB-DESIGN-2016-01.

[53] R. Acciarri et al., Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE), `arXiv:1601.02984`, (2016). FERMILAB-DESIGN-2016-04.

[54] K. Abe et al., Physics potential of a long-baseline neutrino oscillation experiment using a J-PARC neutrino beam and Hyper-Kamiokande, *PTEP.* **2015**, 053C02, (2015).

[55] K. Abe et al., Physics Potentials with the Second Hyper-Kamiokande Detector in Korea, `arXiv:1611.06118`, (2016).

[56] F. An et al., Neutrino Physics with JUNO, *J. Phys.* **G43**(3), 030401, (2016).

[57] S.-B. Kim, New results from RENO and prospects with RENO-50, *Nucl. Part. Phys. Proc.* **265-266**, 93–98, (2015).

[58] M. G. Aartsen et al., Letter of Intent: The Precision IceCube Next Generation Upgrade (PINGU), `arXiv:1401.2046`, (2014).

[59] S. Adrian-Martinez et al., Letter of intent for KM3NeT 2.0, *J. Phys.* **G43**(8), 084001, (2016).

[60] N. Cabibbo, Time Reversal Violation in Neutrino Oscillation, *Phys. Lett.* **B72**, 333–335, (1978).

[61] V. D. Barger, K. Whisnant, and R. J. N. Phillips, CP Violation in Three Neutrino Oscillations, *Phys. Rev. Lett.* **45**, 2084, (1980).

[62] P. I. Krastev and S. T. Petcov, Resonance Amplification and T Violation Effects in Three Neutrino Oscillations in the Earth, *Phys. Lett.* **B205**, 84–92, (1988).

[63] C. Jarlskog, Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Violation, *Phys. Rev. Lett.* **55**, 1039, (1985).

[64] P. Adamson et al., Combined analysis of ν_μ disappearance and $\nu_\mu \to \nu_e$ appearance in MINOS using accelerator and atmospheric neutrinos, *Phys. Rev. Lett.* **112**, 191801, (2014).

[65] K. Abe et al., Letter of Intent: The Hyper-Kamiokande Experiment – Detector Design and Physics Potential, `arXiv:1109.3262`, (2011).

[66] L. Wolfenstein, Neutrino Oscillations in Matter, *Phys. Rev.* **D17**, 2369–2374, (1978).

[67] S. P. Mikheev and A. Yu. Smirnov, Resonance Amplification of Oscillations in Matter and Spectroscopy of Solar Neutrinos, *Sov. J. Nucl. Phys.* **42**, 913–917, (1985). [Yad. Fiz. 42, 1441 (1985)].

[68] V. D. Barger, K. Whisnant, S. Pakvasa, and R. J. N. Phillips, Matter Effects on Three-Neutrino Oscillations, *Phys. Rev.* **D22**, 2718, (1980).

[69] P. Langacker, S. T. Petcov, G. Steigman, and S. Toshev, On the Mikheev-Smirnov-Wolfenstein (MSW) Mechanism of Amplification of Neutrino Oscillations in Matter, *Nucl. Phys.* **B282**, 589–609, (1987).

[70] M. Freund, Analytic approximations for three neutrino oscillation parameters and probabilities in matter, *Phys. Rev.* **D64**, 053003, (2001).

[71] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena, and S. Rigolin, Golden measurements at a neutrino factory, *Nucl. Phys.* **B579**, 17–55, (2000). [Erratum: Nucl. Phys.B593,731(2001)].

[72] A. M. Dziewonski and D. L. Anderson, Preliminary reference earth model, *Phys. Earth Planet. Interiors.* **25**, 297–356, (1981).

[73] S. T. Petcov, Diffractive - like (or parametric resonance - like?) enhancement of the earth (day - night) effect for solar neutrinos crossing the earth core, *Phys. Lett.* **B434**, 321–332, (1998).

[74] S. K. Agarwalla, Y. Kao, and T. Takeuchi, Analytical approximation of the neutrino oscillation matter effects at large θ_{13}, *JHEP.* **04**, 047, (2014).

[75] S. M. Bilenky, S. Pascoli, and S. T. Petcov, Majorana neutrinos, neutrino mass

spectrum, CP violation and neutrinoless double beta decay. 1. The Three neutrino mixing case, *Phys. Rev.* **D64**, 053010, (2001).

[76] S. Pascoli, S. T. Petcov, and L. Wolfenstein, Searching for the CP violation associated with Majorana neutrinos, *Phys. Lett.* **B524**, 319–331, (2002).

[77] V. Barger, S. L. Glashow, P. Langacker, and D. Marfatia, No go for detecting CP violation via neutrinoless double beta decay, *Phys. Lett.* **B540**, 247–251, (2002).

[78] S. Pascoli, S. T. Petcov, and T. Schwetz, The Absolute neutrino mass scale, neutrino mass spectrum, Majorana CP-violation and neutrinoless double-beta decay, *Nucl. Phys.* **B734**, 24–49, (2006).

[79] A. de Gouvea, B. Kayser, and R. N. Mohapatra, Manifest CP violation from Majorana phases, *Phys. Rev.* **D67**, 053004, (2003).

[80] W. Rodejohann, Neutrinoless double beta decay and neutrino physics, *J. Phys.* **G39**, 124008, (2012).

[81] H. Päs and W. Rodejohann, Neutrinoless Double Beta Decay, *New J. Phys.* **17**(11), 115010, (2015).

[82] J. D. Vergados, H. Ejiri, and F. Šimkovic, Neutrinoless double beta decay and neutrino mass, *Int. J. Mod. Phys.* **E25**(11), 1630007, (2016).

[83] L. Wolfenstein, CP Properties of Majorana Neutrinos and Double beta Decay, *Phys. Lett.* **B107**, 77–79, (1981).

[84] S. M. Bilenky, N. P. Nedelcheva, and S. T. Petcov, Some Implications of the CP Invariance for Mixing of Majorana Neutrinos, *Nucl. Phys.* **B247**, 61–69, (1984).

[85] B. Kayser, CPT, CP, and C Phases and their Effects in Majorana Particle Processes, *Phys. Rev.* **D30**, 1023, (1984).

[86] S. M. Bilenky, S. Pascoli, and S. T. Petcov, Majorana neutrinos, neutrino mass spectrum, CP violation and neutrinoless double beta decay. 2. Mixing of four neutrinos, *Phys. Rev.* **D64**, 113003, (2001).

[87] J. Barry, W. Rodejohann, and H. Zhang, Light Sterile Neutrinos: Models and Phenomenology, *JHEP.* **07**, 091, (2011).

[88] I. Girardi, A. Meroni, and S. T. Petcov, Neutrinoless Double Beta Decay in the Presence of Light Sterile Neutrinos, *JHEP.* **11**, 146, (2013).

[89] M. Blennow, E. Fernandez-Martinez, J. López-Pavón, and J. Menendez, Neutrinoless double beta decay in seesaw models, *JHEP.* **07**, 096, (2010).

[90] J. López-Pavón, E. Molinaro, and S. T. Petcov, Radiative Corrections to Light Neutrino Masses in Low Scale Type I Seesaw Scenarios and Neutrinoless Double Beta Decay, *JHEP.* **11**, 030, (2015).

[91] F. F. Deppisch, M. Hirsch, and H. Pas, Neutrinoless Double Beta Decay and Physics Beyond the Standard Model, *J. Phys.* **G39**, 124007, (2012).

[92] F. F. Deppisch, P. S. Bhupal Dev, and A. Pilaftsis, Neutrinos and Collider Physics, *New J. Phys.* **17**(7), 075019, (2015).

[93] M. Agostini. GERDA Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.

[94] D. Waters. NEMO-3 Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.

[95] L. Canonica. CUORE Collaboration. Talk given at the XXVII International Conference on Neutrino Physics and Astrophysics (Neutrino 2016), London, United Kingdom, 4-9 July, 2016.

[96] A. Gando et al., Search for Majorana Neutrinos near the Inverted Mass Hierarchy Region with KamLAND-Zen, *Phys. Rev. Lett.* **117**(8), 082503, (2016). [Addendum:

Phys. Rev. Lett.117,no.10,109903(2016)].

[97] M. Auger et al., Search for Neutrinoless Double-Beta Decay in ^{136}Xe with EXO-200, *Phys. Rev. Lett.* **109**, 032505, (2012).

[98] S. M. Bilenky, C. Giunti, C. W. Kim, and S. T. Petcov, Short baseline neutrino oscillations and neutrinoless double beta decay in schemes with an inverted mass spectrum, *Phys. Rev.* **D54**, 4432–4444, (1996).

[99] A. Faessler, G. L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno, and F. Simkovic, QRPA uncertainties and their correlations in the analysis of 0 nu beta beta decay, *Phys. Rev.* **D79**, 053001, (2009).

[100] S. M. Bilenky and S. T. Petcov, Nuclear matrix elements of $0\nu\beta\beta$ decay: Possible test of the calculations, `arXiv:hep-ph/0405237`, (2004).

[101] A. Faessler, A. Meroni, S. T. Petcov, F. Simkovic, and J. Vergados, Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$ Decay, *Phys. Rev.* **D83**, 113003, (2011).

[102] A. Meroni, S. T. Petcov, and F. Simkovic, Multiple CP non-conserving mechanisms of $(\beta\beta)_{0\nu}$-decay and nuclei with largely different nuclear matrix elements, *JHEP.* **02**, 025, (2013).

[103] S. Pascoli and S. T. Petcov, Majorana Neutrinos, Neutrino Mass Spectrum and the $|\langle m\rangle| \sim 10^{-3}$ eV Frontier in Neutrinoless Double Beta Decay, *Phys. Rev.* **D77**, 113003, (2008).

[104] P. Minkowski, $\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?, *Phys. Lett.* **B67**, 421–428, (1977).

[105] T. Yanagida, Horizontal Symmetry and Masses of Neutrinos, *Conf. Proc.* **C7902131**, 95–99, (1979).

[106] M. Gell-Mann, P. Ramond, and R. Slansky, Complex Spinors and Unified Theories, *Conf. Proc.* **C790927**, 315–321, (1979).

[107] R. N. Mohapatra and G. Senjanovic, Neutrino Mass and Spontaneous Parity Violation, *Phys. Rev. Lett.* **44**, 912, (1980).

[108] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe, *Phys. Lett.* **B155**, 36, (1985).

[109] M. Fukugita and T. Yanagida, Baryogenesis Without Grand Unification, *Phys. Lett.* **B174**, 45–47, (1986).

[110] S. Biondini, D. Bödeker, N. Brambilla, M. Garny, J. Ghiglieri, A. Hohenegger, M. Laine, S. Mendizabal, P. Millington, A. Salvio, and A. Vairo, Status of rates and rate equations for thermal leptogenesis, `arXiv:1711.02864`, (2017).

[111] S. Davidson, E. Nardi, and Y. Nir, Leptogenesis, *Phys. Rept.* **466**, 105–177, (2008).

[112] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, Towards a complete theory of thermal leptogenesis in the SM and MSSM, *Nucl. Phys.* **B685**, 89–149, (2004).

[113] L. Covi, E. Roulet, and F. Vissani, CP violating decays in leptogenesis scenarios, *Phys. Lett.* **B384**, 169–174, (1996).

[114] C. S. Fong, E. Nardi, and A. Riotto, Leptogenesis in the Universe, *Adv. High Energy Phys.* **2012**, 158303, (2012).

[115] S. Blanchet and P. Di Bari, The minimal scenario of leptogenesis, *New J. Phys.* **14**, 125012, (2012).

[116] E. Molinaro, S. T. Petcov, T. Shindou, and Y. Takanishi, Effects of Lightest Neutrino Mass in Leptogenesis, *Nucl. Phys.* **B797**, 93–116, (2008).

[117] R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Baryogenesis through leptogenesis, *Nucl. Phys.* **B575**, 61–77, (2000).

[118] A. Abada, S. Davidson, F.-X. Josse-Michaux, M. Losada, and A. Riotto, Flavor issues in leptogenesis, *JCAP*. **0604**, 004, (2006).
[119] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada, and A. Riotto, Flavour Matters in Leptogenesis, *JHEP*. **09**, 010, (2006).
[120] E. Nardi, Y. Nir, E. Roulet, and J. Racker, The Importance of flavor in leptogenesis, *JHEP*. **01**, 164, (2006).
[121] P. S. B. Dev, P. Di Bari, B. Garbrecht, S. Lavignac, P. Millington, and D. Teresi, Flavor effects in leptogenesis, `arXiv:1711.02861`, (2017).
[122] G. C. Branco, T. Morozumi, B. M. Nobre, and M. N. Rebelo, A Bridge between CP violation at low-energies and leptogenesis, *Nucl. Phys.* **B617**, 475–492, (2001).
[123] M. N. Rebelo, Leptogenesis without CP violation at low-energies, *Phys. Rev.* **D67**, 013008, (2003).
[124] S. Pascoli, S. T. Petcov, and W. Rodejohann, On the connection of leptogenesis with low-energy CP violation and LFV charged lepton decays, *Phys. Rev.* **D68**, 093007, (2003).
[125] S. Davidson, J. Garayoa, F. Palorini, and N. Rius, Insensitivity of flavoured leptogenesis to low energy CP violation, *Phys. Rev. Lett.* **99**, 161801, (2007).
[126] S. Davidson, J. Garayoa, F. Palorini, and N. Rius, CP Violation in the SUSY Seesaw: Leptogenesis and Low Energy, *JHEP*. **09**, 053, (2008).
[127] S. Pascoli, S. T. Petcov, and A. Riotto, Connecting low energy leptonic CP-violation to leptogenesis, *Phys. Rev.* **D75**, 083511, (2007).
[128] S. Pascoli, S. T. Petcov, and A. Riotto, Leptogenesis and Low Energy CP Violation in Neutrino Physics, *Nucl. Phys.* **B774**, 1–52, (2007).
[129] E. Molinaro and S. T. Petcov, The Interplay Between the "Low" and "High" Energy CP-Violation in Leptogenesis, *Eur. Phys. J.* **C61**, 93–109, (2009).
[130] E. Molinaro and S. T. Petcov, A Case of Subdominant/Suppressed "High Energy" Contribution to the Baryon Asymmetry of the Universe in Flavoured Leptogenesis, *Phys. Lett.* **B671**, 60–65, (2009).
[131] S. Antusch and A. M. Teixeira, Towards constraints on the SUSY seesaw from flavour-dependent leptogenesis, *JCAP*. **0702**, 024, (2007).
[132] S. Blanchet and P. Di Bari, Flavor effects on leptogenesis predictions, *JCAP*. **0703**, 018, (2007).
[133] R. N. Mohapatra and S. Nasri, Leptogenesis and $\mu - \tau$ symmetry, *Phys. Rev.* **D71**, 033001, (2005).
[134] R. N. Mohapatra, S. Nasri, and H.-B. Yu, Leptogenesis, $\mu - \tau$ symmetry and θ_{13}, *Phys. Lett.* **B615**, 231–239, (2005).
[135] C. Hagedorn, E. Molinaro, and S. T. Petcov, Majorana Phases and Leptogenesis in See-Saw Models with A_4 Symmetry, *JHEP*. **09**, 115, (2009).
[136] R. N. Mohapatra and C. C. Nishi, Implications of $\mu - \tau$ flavored CP symmetry of leptons, *JHEP*. **08**, 092, (2015).
[137] C. Hagedorn and E. Molinaro, Flavor and CP symmetries for leptogenesis and $0\nu\beta\beta$ decay, *Nucl. Phys.* **B919**, 404–469, (2017).
[138] C. S. Lam, Symmetry of Lepton Mixing, *Phys. Lett.* **B656**, 193–198, (2007).
[139] A. Blum, C. Hagedorn, and M. Lindner, Fermion Masses and Mixings from Dihedral Flavor Symmetries with Preserved Subgroups, *Phys. Rev.* **D77**, 076004, (2008).
[140] D. Hernandez and A. Yu. Smirnov, Lepton mixing and discrete symmetries, *Phys. Rev.* **D86**, 053014, (2012).
[141] W. Grimus and L. Lavoura, Leptogenesis in seesaw models with a twofold degenerate neutrino Dirac mass matrix, *J. Phys.* **G30**(9), 1073–1088, (2004).
[142] E. E. Jenkins and A. V. Manohar, Tribimaximal Mixing, Leptogenesis and θ_{13}, *Phys.*

Lett. **B668**, 210–215, (2008).
[143] E. Bertuzzo, P. Di Bari, F. Feruglio, and E. Nardi, Flavor symmetries, leptogenesis and the absolute neutrino mass scale, *JHEP.* **11**, 036, (2009).
[144] D. Aristizabal Sierra, F. Bazzocchi, I. de Medeiros Varzielas, L. Merlo, and S. Morisi, Tri-Bimaximal Lepton Mixing and Leptogenesis, *Nucl. Phys.* **B827**, 34–58, (2010).
[145] W. Grimus and L. Lavoura, A non-standard CP transformation leading to maximal atmospheric neutrino mixing, *Phys. Lett.* **B579**, 113–122, (2004).
[146] P. Chen, G.-J. Ding, and S. F. King, Leptogenesis and residual CP symmetry, *JHEP.* **03**, 206, (2016).
[147] J. Gehrlein, S. T. Petcov, M. Spinrath, and X. Zhang, Leptogenesis in an SU(5) $\times$ A_5 Golden Ratio Flavour Model, *Nucl. Phys.* **B896**, 311–329, (2015).
[148] G.-J. Ding, S. F. King, and T. Neder, Generalised CP and $\Delta(6n^2)$ family symmetry in semi-direct models of leptons, *JHEP.* **12**, 007, (2014).
[149] S. F. King and T. Neder, Lepton mixing predictions including Majorana phases from $\Delta(6n^2)$ flavour symmetry and generalised CP, *Phys. Lett.* **B736**, 308–316, (2014).
[150] S. T. Petcov, Predicting the values of the leptonic CP violation phases in theories with discrete flavour symmetries, *Nucl. Phys.* **B892**, 400–428, (2015).
[151] T. Fukuyama and H. Nishiura, Mass matrix of Majorana neutrinos, `arXiv:hep-ph/9702253`, (1997).
[152] R. N. Mohapatra and S. Nussinov, Bimaximal neutrino mixing and neutrino mass matrix, *Phys. Rev.* **D60**, 013002, (1999).
[153] C. S. Lam, Mass Independent Textures and Symmetry, *Phys. Rev.* **D74**, 113004, (2006).
[154] P. F. Harrison, D. H. Perkins, and W. G. Scott, Tri-bimaximal mixing and the neutrino oscillation data, *Phys. Lett.* **B530**, 167, (2002).
[155] Z.-Z. Xing, Nearly tri-bimaximal neutrino mixing and CP violation, *Phys. Lett.* **B533**, 85–93, (2002).
[156] G. Altarelli and F. Feruglio, Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions, *Nucl. Phys.* **B720**, 64–88, (2005).
[157] I. de Medeiros Varzielas, S. F. King, and G. G. Ross, Tri-bimaximal neutrino mixing from discrete subgroups of SU(3) and SO(3) family symmetry, *Phys. Lett.* **B644**, 153–157, (2007).
[158] G. Altarelli and F. Feruglio, Tri-bimaximal neutrino mixing, A_4 and the modular symmetry, *Nucl. Phys.* **B741**, 215–235, (2006).
[159] X.-G. He, Y.-Y. Keum, and R. R. Volkas, A_4 flavor symmetry breaking scheme for understanding quark and neutrino mixing angles, *JHEP.* **04**, 039, (2006).
[160] C. S. Lam, Determining Horizontal Symmetry from Neutrino Mixing, *Phys. Rev. Lett.* **101**, 121602, (2008).
[161] R. de Adelhart Toorop, F. Feruglio, and C. Hagedorn, Finite Modular Groups and Lepton Mixing, *Nucl. Phys.* **B858**, 437–467, (2012).
[162] S. F. King, T. Neder, and A. J. Stuart, Lepton mixing predictions from $\Delta(6n^2)$ family Symmetry, *Phys. Lett.* **B726**, 312–315, (2013).
[163] C. Hagedorn, A. Meroni, and L. Vitale, Mixing patterns from the groups $\Sigma(n\phi)$, *J. Phys.* **A47**, 055201, (2014).
[164] R. M. Fonseca and W. Grimus, Classification of lepton mixing matrices from finite residual symmetries, *JHEP.* **09**, 033, (2014).
[165] G. Altarelli and F. Feruglio, Discrete Flavor Symmetries and Models of Neutrino Mixing, *Rev. Mod. Phys.* **82**, 2701–2729, (2010).
[166] S. F. King, A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, Neutrino Mass and Mixing: from Theory to Experiment, *New J. Phys.* **16**, 045018, (2014).

[167] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Non-Abelian Discrete Symmetries in Particle Physics, *Prog. Theor. Phys. Suppl.* **183**, 1–163, (2010).

[168] W. Grimus and P. O. Ludl, Finite flavour groups of fermions, *J. Phys.* **A45**, 233001, (2012).

[169] I. Girardi, S. T. Petcov, and A. V. Titov, Determining the Dirac CP Violation Phase in the Neutrino Mixing Matrix from Sum Rules, *Nucl. Phys.* **B894**, 733–768, (2015).

[170] P. Ballett, S. F. King, C. Luhn, S. Pascoli, and M. A. Schmidt, Testing solar lepton mixing sum rules in neutrino oscillation experiments, *JHEP.* **12**, 122, (2014).

[171] I. Girardi, S. T. Petcov, and A. V. Titov, Predictions for the Leptonic Dirac CP Violation Phase: a Systematic Phenomenological Analysis, *Eur. Phys. J.* **C75**, 345, (2015).

[172] I. Girardi, S. T. Petcov, A. J. Stuart, and A. V. Titov, Leptonic Dirac CP Violation Predictions from Residual Discrete Symmetries, *Nucl. Phys.* **B902**, 1–57, (2016).

[173] I. Girardi, S. T. Petcov, and A. V. Titov, Predictions for the Majorana CP Violation Phases in the Neutrino Mixing Matrix and Neutrinoless Double Beta Decay, *Nucl. Phys.* **B911**, 754–804, (2016).

[174] F. Feruglio, C. Hagedorn, and R. Ziegler, Lepton Mixing Parameters from Discrete and CP Symmetries, *JHEP.* **07**, 027, (2013).

[175] M. Holthausen, M. Lindner, and M. A. Schmidt, CP and Discrete Flavour Symmetries, *JHEP.* **04**, 122, (2013).

[176] M.-C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz, and A. Trautner, CP Violation from Finite Groups, *Nucl. Phys.* **B883**, 267–305, (2014).

[177] W. Grimus and M. N. Rebelo, Automorphisms in gauge theories and the definition of CP and P, *Phys. Rept.* **281**, 239–308, (1997).

[178] P. F. Harrison and W. G. Scott, Symmetries and generalizations of tri-bimaximal neutrino mixing, *Phys. Lett.* **B535**, 163–169, (2002).

[179] C. Hagedorn, A. Meroni, and E. Molinaro, Lepton mixing from $\Delta(3n^2)$ and $\Delta(6n^2)$ and CP, *Nucl. Phys.* **B891**, 499–557, (2015).

[180] A. Di Iura, C. Hagedorn, and D. Meloni, Lepton mixing from the interplay of the alternating group A_5 and CP, *JHEP.* **08**, 037, (2015).

[181] C.-C. Li and G.-J. Ding, Lepton Mixing in A_5 Family Symmetry and Generalized CP, *JHEP.* **05**, 100, (2015).

[182] P. Ballett, S. Pascoli, and J. Turner, Mixing angle and phase correlations from A_5 with generalized CP and their prospects for discovery, *Phys. Rev.* **D92**(9), 093008, (2015).

[183] S.-J. Rong, Lepton mixing patterns from the group $\Sigma(36 \times 3)$ with a generalized CP transformation, *Phys. Rev.* **D95**(7), 076014, (2017).

[184] K. S. Babu, C. N. Leung, and J. T. Pantaleone, Renormalization of the neutrino mass operator, *Phys. Lett.* **B319**, 191–198, (1993).

[185] S. Antusch, M. Drees, J. Kersten, M. Lindner, and M. Ratz, Neutrino mass operator renormalization revisited, *Phys. Lett.* **B519**, 238–242, (2001).

[186] J. A. Casas and A. Ibarra, Oscillating neutrinos and $\mu \to e\,\gamma$, *Nucl. Phys.* **B618**, 171–204, (2001).

[187] C. Luhn, S. Nasri, and P. Ramond, The Flavor group $\Delta(3n^2)$, *J. Math. Phys.* **48**, 073501, (2007).

[188] J. A. Escobar and C. Luhn, The Flavor Group $\Delta(6n^2)$, *J. Math. Phys.* **50**, 013524, (2009).

[189] G. C. Branco, L. Lavoura, and M. N. Rebelo, Majorana Neutrinos and CP Violation in the Leptonic Sector, *Phys. Lett.* **B180**, 264–268, (1986).

[190] J. F. Nieves and P. B. Pal, Minimal Rephasing Invariant CP Violating Parameters With Dirac and Majorana Fermions, *Phys. Rev.* **D36**, 315, (1987).
[191] M. Magg and C. Wetterich, Neutrino Mass Problem and Gauge Hierarchy, *Phys. Lett.* **B94**, 61–64, (1980).
[192] J. Schechter and J. W. F. Valle, Neutrino Masses in SU(2) x U(1) Theories, *Phys. Rev.* **D22**, 2227, (1980).
[193] R. N. Mohapatra and G. Senjanovic, Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation, *Phys. Rev.* **D23**, 165, (1981).
[194] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Seesaw Neutrino Masses Induced by a Triplet of Leptons, *Z. Phys.* **C44**, 441, (1989).
[195] F. Bazzocchi, L. Merlo, and S. Morisi, Phenomenological Consequences of See-Saw in S_4 Based Models, *Phys. Rev.* **D80**, 053003, (2009).