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## ERRATUM

# Erratum: Asymptotic work statistics of periodically driven Ising chains

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- 1 The demonstration in appendix C, starting from the fourth line after equation C.5 to the end, is incorrect. This does not change the main results, but modifies some details. Here is the corrected version:

We see, from equation (7), that the symmetry relation

$$\mathbb{H}_{-k}(t) = \sigma_z \mathbb{H}_k(t) \sigma_z$$

is valid at all times. Because  $\sigma_z$  is a time-independent unitary transformation, equation (C.4) implies that  $\mathbb{H}_{-k}^F = \sigma_z \mathbb{H}_k^F \sigma_z$ . Because of equation (C.5), and the relations  $\text{Tr}[\mathbb{H}_k(t)] = 0$ ,  $\sigma_z^3 = \sigma_z$ ,  $\sigma_z \sigma_x \sigma_z = -\sigma_x$ ,  $\sigma_z \sigma_y \sigma_z = -\sigma_y$ , we can write the following second-order-in- $k$  expansion<sup>6</sup> of  $\mathbb{H}_k^F$

<sup>6</sup> The vanishing of the trace of  $\mathbb{H}_k^F$  at any  $k$  comes from the vanishing of the trace of  $\mathbb{H}_k(t)$  and the formula  $\text{Tr}[\mathbb{H}_k^F] = \frac{1}{\tau} \int_0^\tau \text{Tr}[\mathbb{H}_k(t)] d\tau$ , which is a corollary of the Liouville's theorem [49].



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$$\mathbb{H}_{\text{small } k}^F = \begin{pmatrix} \tilde{h} + a_z k^2 & (a_x - i a_y) k \\ (a_x + i a_y) k & -\tilde{h} - a_z k^2 \end{pmatrix} + \mathcal{O}(k^3). \quad (\text{C.6})$$

In general the coefficients  $a_x$ ,  $a_y$  and  $a_z$  are non-vanishing; they can vanish in some cases, giving rise to interesting phenomena which we will discuss later. Whenever the resonance condition equation (C.3) is *not* fulfilled (hence  $\tilde{h} \neq 0$ ), the second-order-in- $k$  expansion of Floquet modes (expressed in the basis  $\mathcal{B} = \{|0\rangle, \hat{c}_k^\dagger \hat{c}_{-k}^\dagger |0\rangle\}$ ) is

$$\left[ |\phi_{\text{small } k}^+ \rangle \right]_{\mathcal{B}} = \begin{pmatrix} 1 - \frac{1}{8} \frac{a_y^2 + a_x^2}{\tilde{h}^2} k^2 \\ -\frac{1}{2} \frac{a_x + i a_y}{\tilde{h}} k \end{pmatrix} \quad \text{and} \quad \left[ |\phi_{\text{small } k}^- \rangle \right]_{\mathcal{B}} = \begin{pmatrix} \frac{1}{2} \frac{a_x - i a_y}{\tilde{h}} k \\ 1 - \frac{1}{8} \frac{a_x^2 + a_y^2}{\tilde{h}^2} k^2 \end{pmatrix}, \quad (\text{C.7})$$

which applies to the case  $\tilde{h} > 0$ ; these two states should be exchanged if  $\tilde{h} < 0$ . Moving now to the initial Hamiltonian ground state  $|\psi_k^{\text{gs}}\rangle$ , the diagonalization of equation (7) (with  $h(t) = h_i$ ) immediately gives (for  $h_i > h_c$ )

$$\left[ |\psi_{\text{small } k}^{\text{gs}} \rangle \right]_{\mathcal{B}} = \begin{pmatrix} \frac{i k}{2(h_i - h_c)} \\ 1 - \frac{k^2}{8(h_i - h_c)^2} \end{pmatrix}. \quad (\text{C.8})$$

Hence, for the overlap  $|r_k^+|^2$  we find

$$|r_k^+|^2 = |\langle \phi_k^+ | \psi_k^{\text{gs}} \rangle|^2 = \frac{1}{4} \alpha^2 k^2 + \mathcal{O}(k^3) \quad \text{with} \quad \alpha^2 \equiv \left( \frac{1}{h_i - h_c} + \frac{a_y}{\tilde{h}} \right)^2 + \left( \frac{a_x}{\tilde{h}} \right)^2$$

which is indeed equation (B.6); this formula is valid also in the case  $\tilde{h} < 0$  and  $h_i < h_c$ . If  $\tilde{h} < 0$  and  $h_i > h_c$ , or  $\tilde{h} > 0$  and  $h_i < h_c$ , we find  $|r_k^-|^2 = \frac{\alpha^2}{4} k^2 + \mathcal{O}(k^3)$ , but the crucial ingredient determining equation (A.9) is identical, since  $\xi_k \simeq \alpha^2 k^2$  in both cases (see equation (A.2)).

For the resonant case  $\tilde{h} = 0$  we find

$$\left[ |\phi_{\text{small } k}^+ \rangle \right]_{\mathcal{B}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + \frac{1}{2} \frac{a_z}{\sqrt{a_x^2 + a_y^2}} k \\ \frac{-a_x + i a_y}{\sqrt{a_x^2 + a_y^2}} \left( 1 - \frac{1}{2} \frac{a_z}{\sqrt{a_x^2 + a_y^2}} k \right) \end{pmatrix}$$

which (if  $h_i > h_c$ ) gives rise to

$$|r_k^+|^2 = \frac{1}{2} \left( 1 - \frac{a_z}{\sqrt{a_x^2 + a_y^2}} k \right) + \mathcal{O}(k^2).$$

This is indeed equation (B.5) with  $\beta = a_z/\sqrt{a_x^2 + a_y^2}$ . We notice that this formula is valid if  $a_x - ia_y \neq 0$ . This is generally true, up to special cases where there is coherent destruction of tunnelling (CDT) [48]: here  $a_x = a_y = 0$ , and we fall back to equation (B.6). In the Supplemental material of [28] we show that, in the case of a sinusoidal driving  $h(t) = h_0 + A \cos(\omega_0 t + \phi_0)$ , CDT occurs if  $h_0 = 1$  and  $J_0(2A/\omega_0) = 0$ . More in general, if the resonance condition equation (C.3) is valid for  $l \neq 0$ , we can show—exactly with the same arguments used in [28]—that there is CDT whenever  $J_l(2A/\omega_0) = 0$ .